Efficient Probabilistic Visualization of Local Divergence of 2D Vector Fields with Independent Gaussian Uncertainty

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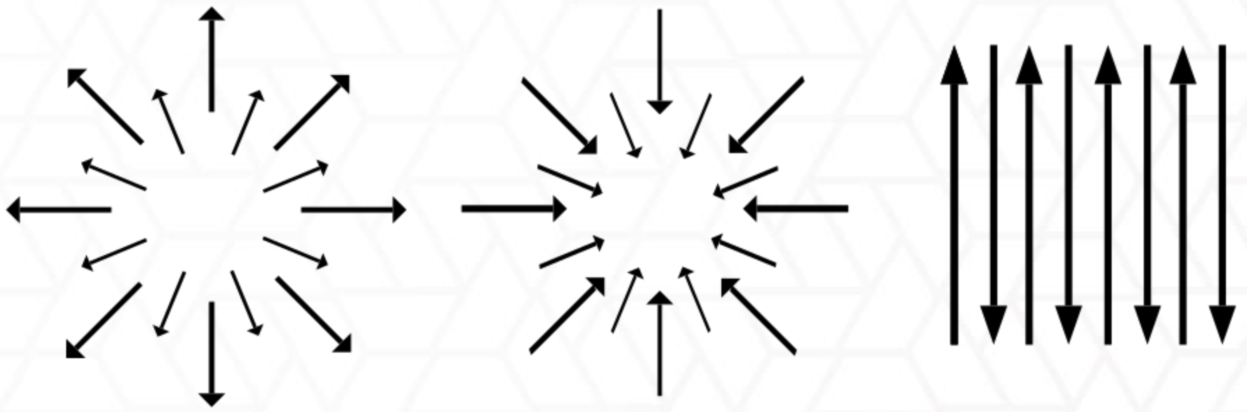
Divergence for flow analysis and diagnostic

- Divergence measures the rate at which the flow spreads (positive divergence) or converges (negative divergence) at a point
- Used understanding fluid behavior across scientific and engineering domains
- Useful for identifying sinks and sources

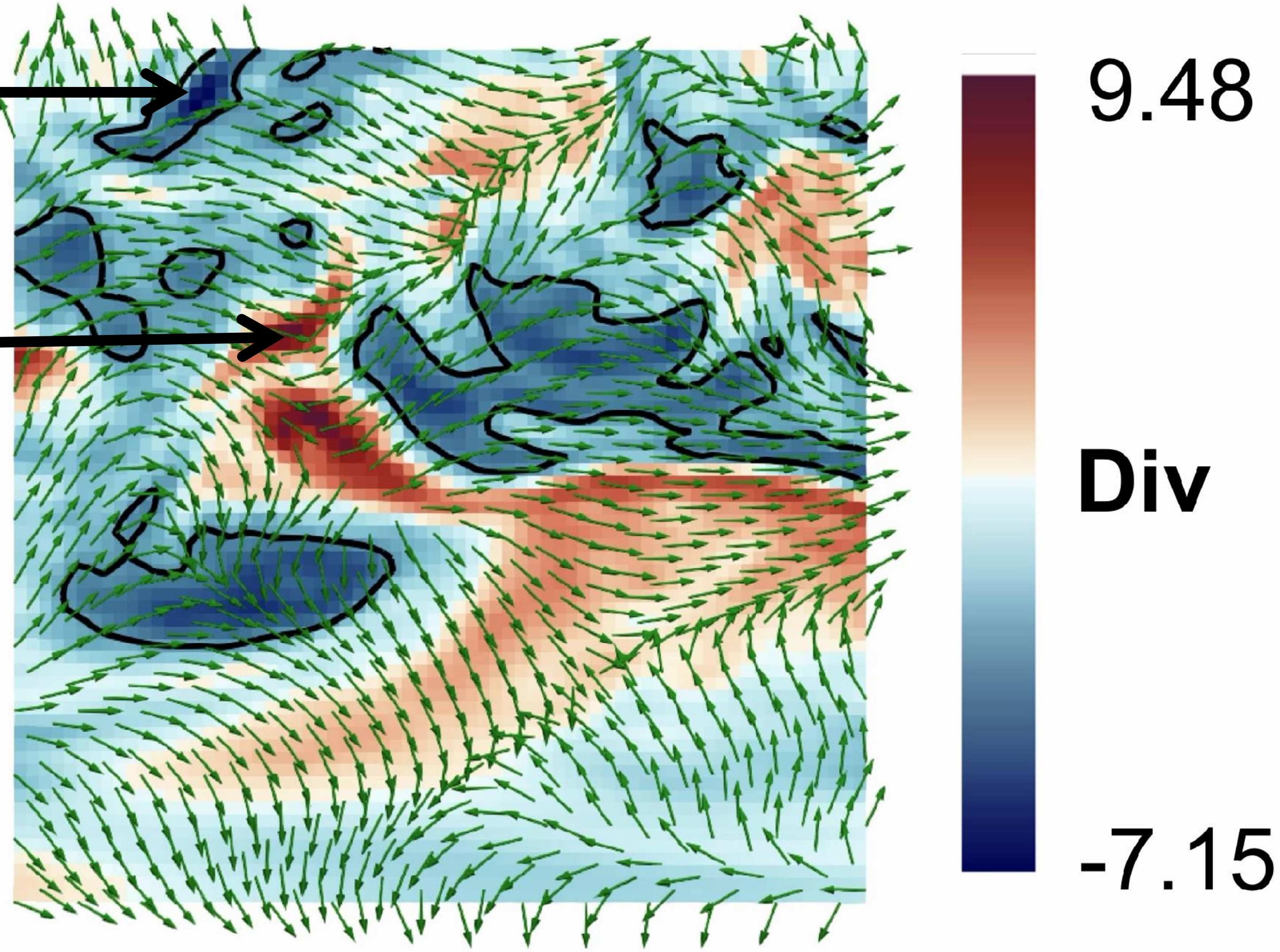


x-component y-component

Vector $\boldsymbol{f} = (u, v)$

Divergence $\nabla \cdot \boldsymbol{f} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}$

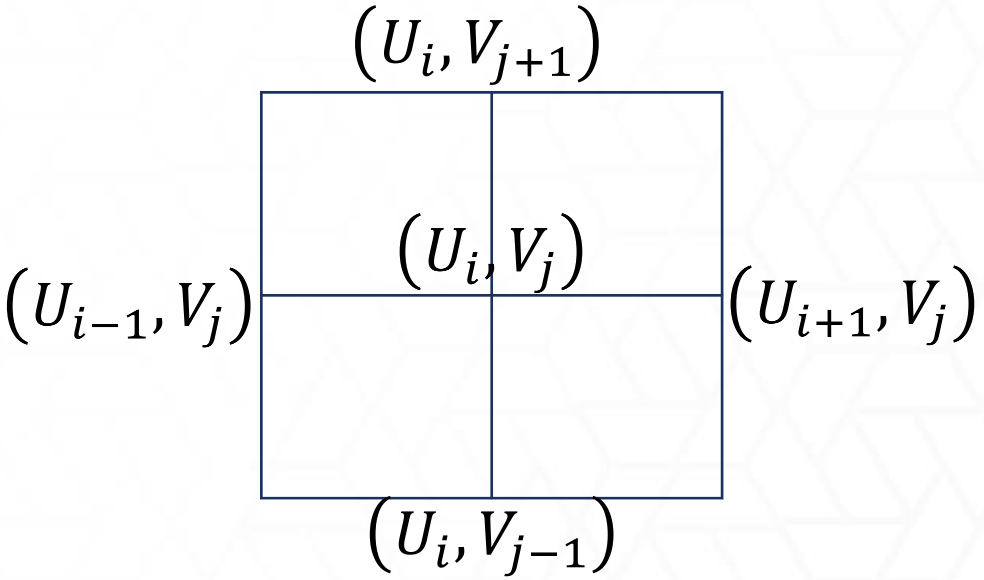
Divergence for flow analysis and diagnostic



Uncertainty in vector field data creates uncertainty about source/sink positions!

Probabilistic Divergence

$$\boldsymbol{f} = (u, v) \qquad \nabla \cdot \boldsymbol{f} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \longrightarrow \boxed{\nabla \cdot \boldsymbol{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}}$$



mean

variance

$$U_i \sim \mathcal{N}\left(\mu_i, \sigma_i^2\right)$$

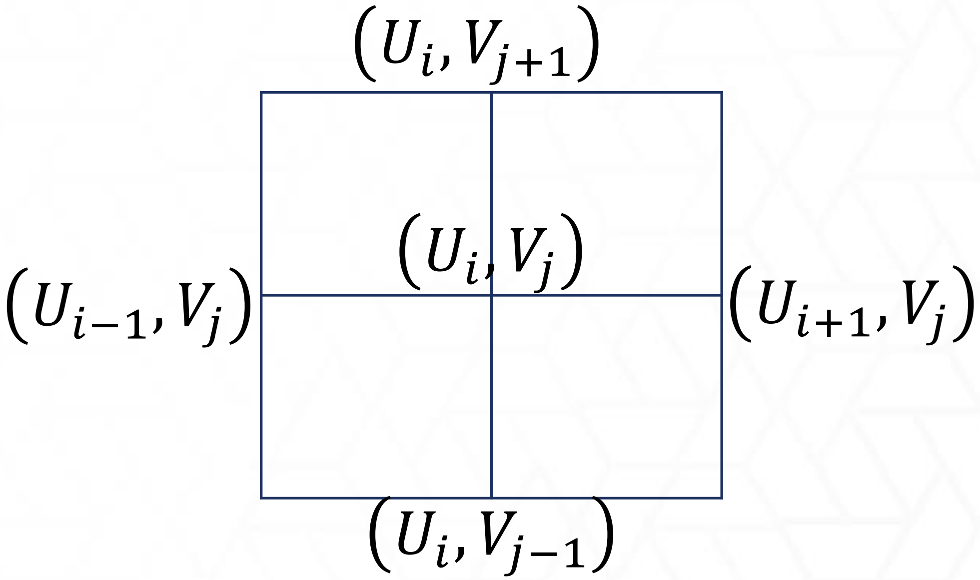
$$V_j \sim \mathcal{N}\left(\mu_j, \sigma_j^2\right)$$

Independent Gaussians

$$\boxed{\nabla \cdot \boldsymbol{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}} \longrightarrow \nabla \cdot \boldsymbol{F}_{i,j} = \frac{U_{i+1} - U_{i-1}}{2\Delta x} + \frac{V_{i+1} - V_{i-1}}{2\Delta y}$$

Probabilistic Divergence

$$\boxed{\nabla \cdot \boldsymbol{F} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}} \longrightarrow \nabla \cdot \boldsymbol{F}_{i,j} = \frac{U_{i+1} - U_{i-1}}{2\Delta x} + \frac{V_{i+1} - V_{i-1}}{2\Delta y}$$



$$\overbrace{\mu_{i,j} = \frac{\mu_{i+1}}{2\Delta x} - \frac{\mu_{i-1}}{2\Delta x} + \frac{\mu_{j+1}}{2\Delta y} - \frac{\mu_{j-1}}{2\Delta y}}^{\text{mean}}$$

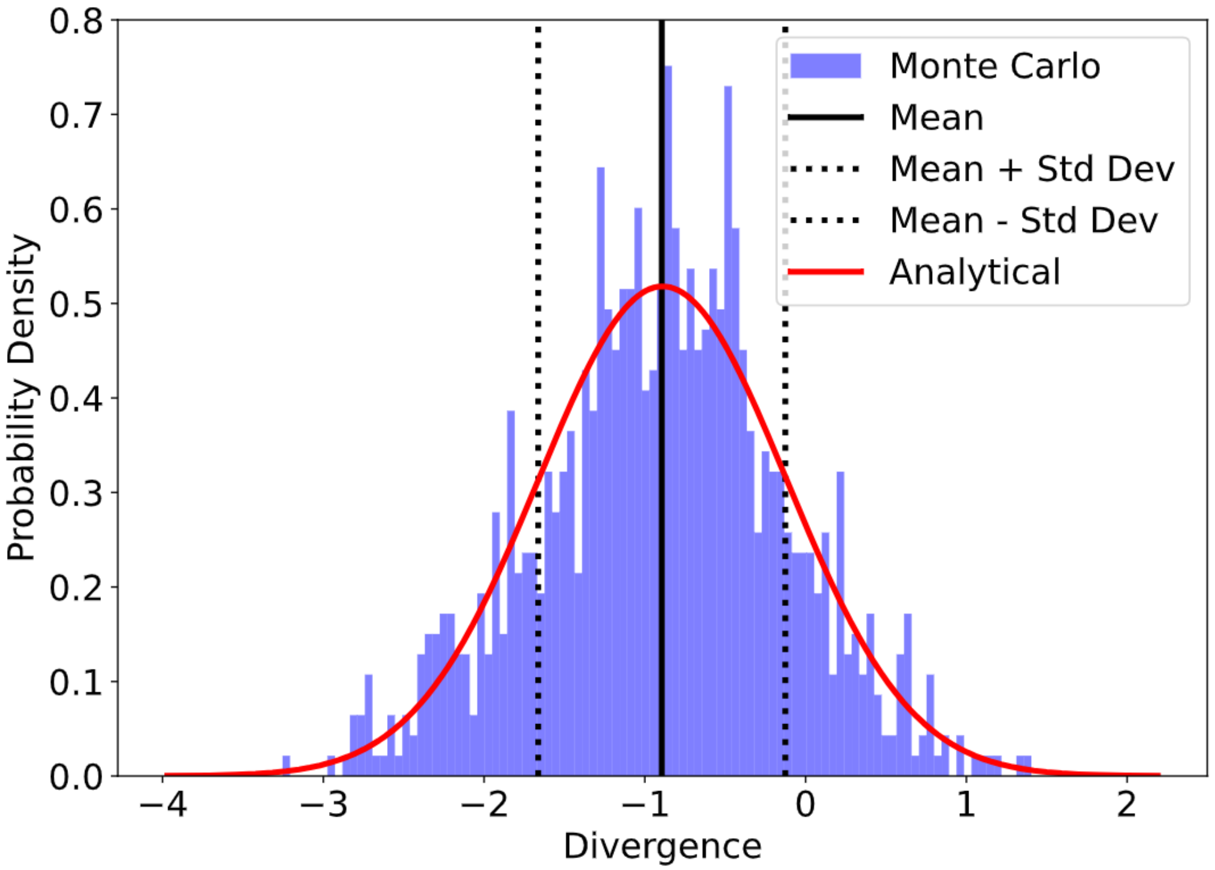
$$\nabla \cdot \boldsymbol{F}_{i,j} = \frac{U_{i+1} - U_{i-1}}{2\Delta x} + \frac{V_{i+1} - V_{i-1}}{2\Delta y} \longrightarrow \boxed{\nabla \cdot \boldsymbol{F}_{i,j} \sim \mathcal{N}(\mu_{i,j}, \sigma_{i,j}^2)}$$

Sum of Gaussians

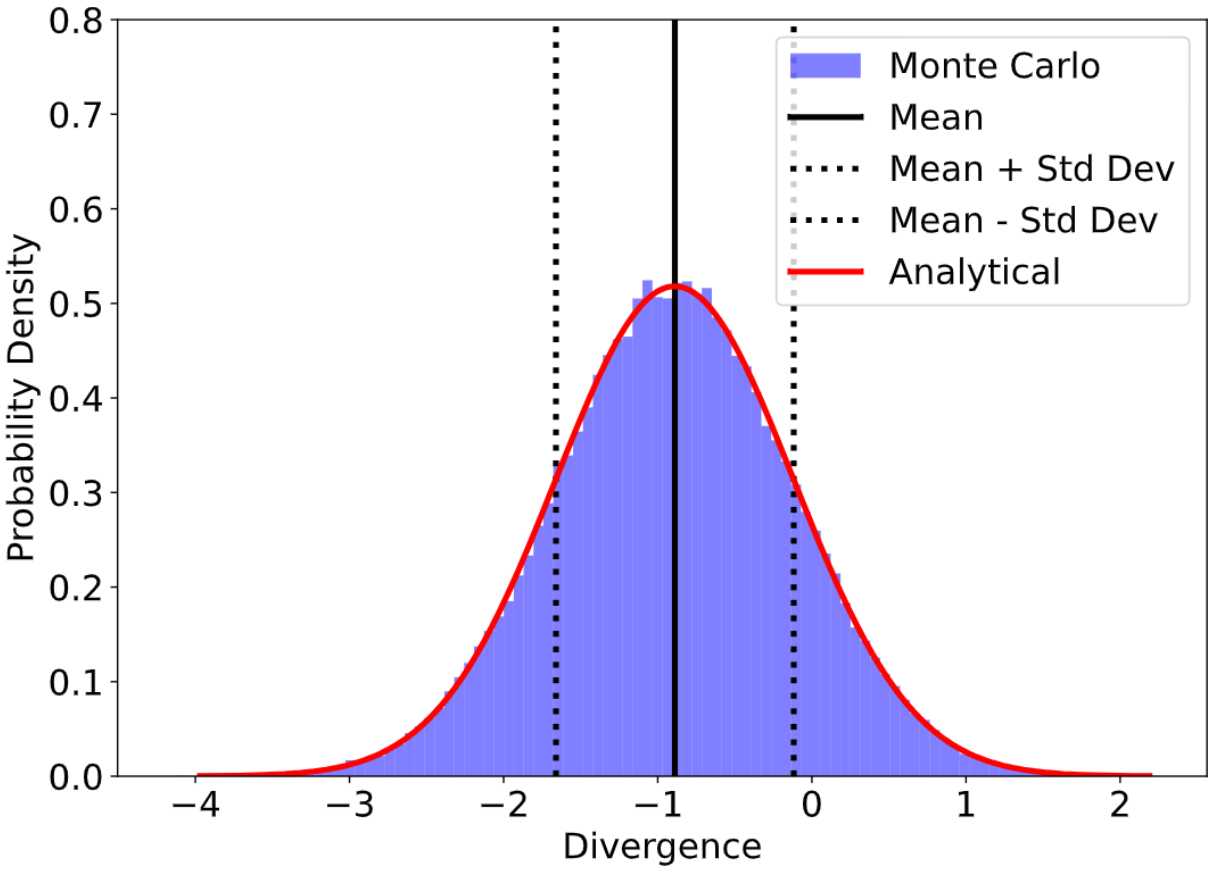
$$\overbrace{\sigma_{i,j}^2 = \left(\frac{\sigma_{i+1}}{2\Delta x}\right)^2 - \left(\frac{\sigma_{i-1}}{2\Delta x}\right)^2 + \left(\frac{\sigma_{j+1}}{2\Delta y}\right)^2 - \left(\frac{\sigma_{j-1}}{2\Delta y}\right)^2}^{\text{variance}}$$

Probabilistic Divergence Validation

1000 samples

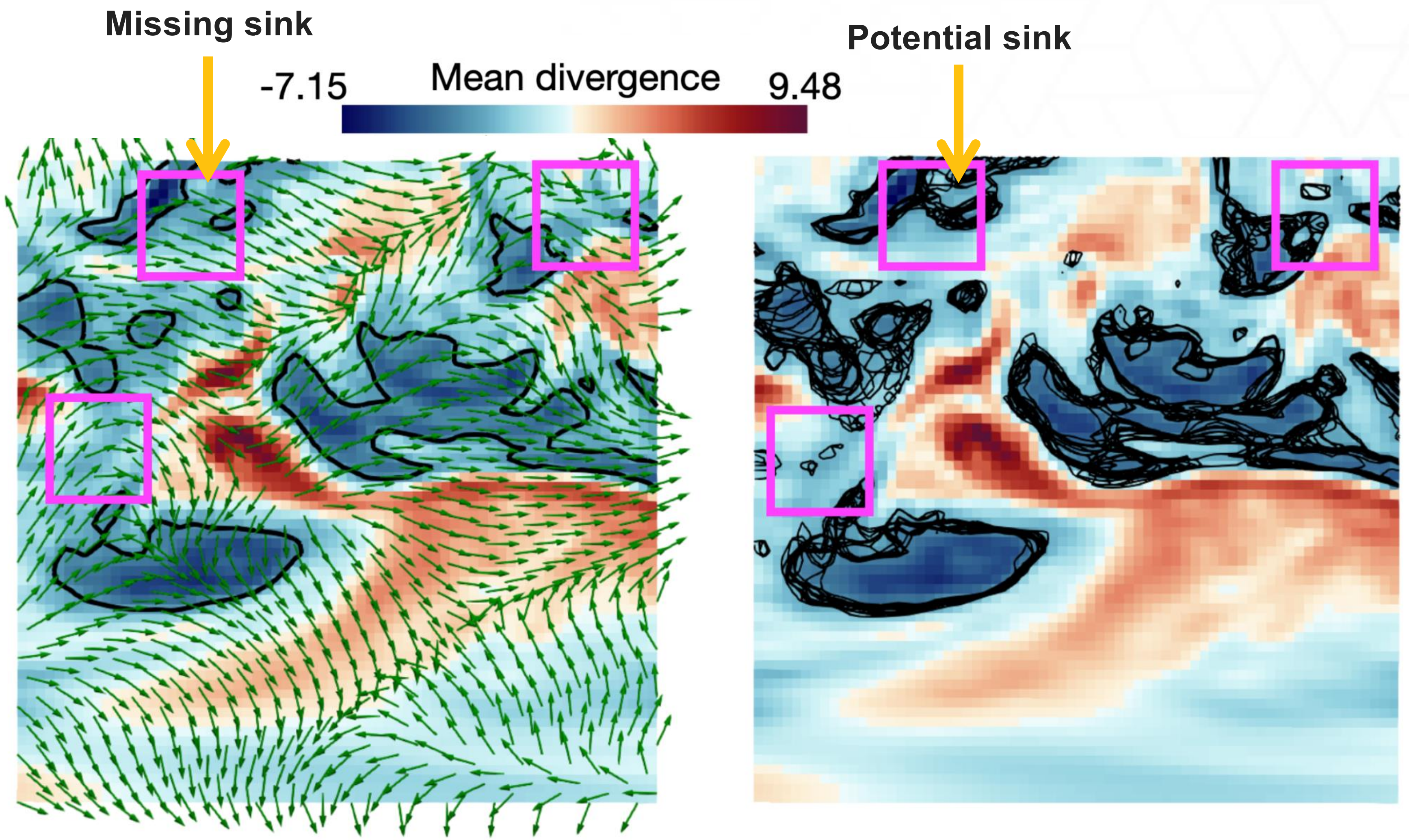


100000 samples



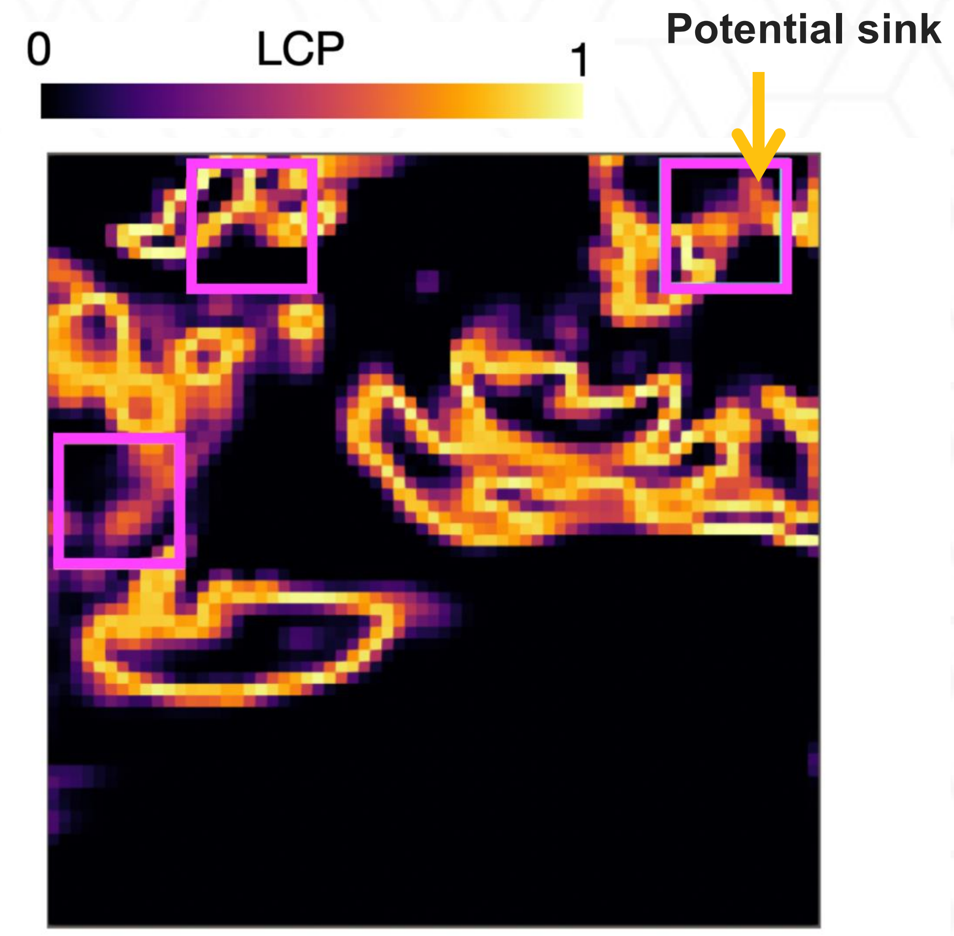
Wind Dataset Example

Pöthkow, K., Weber, B., & Hege, H. C. (2011, June). Probabilistic marching cubes. In *Computer Graphics Forum* (Vol. 30, No. 3, pp. 931-940). Oxford, UK: Blackwell Publishing Ltd.



(a) Mean-field isocontour
(does not depict uncertainty)

(b) Spaghetti plot
(depicts uncertainty, but cluttered)



(c) Probabilistic isocontour
(depicts uncertainty without visual clutter)

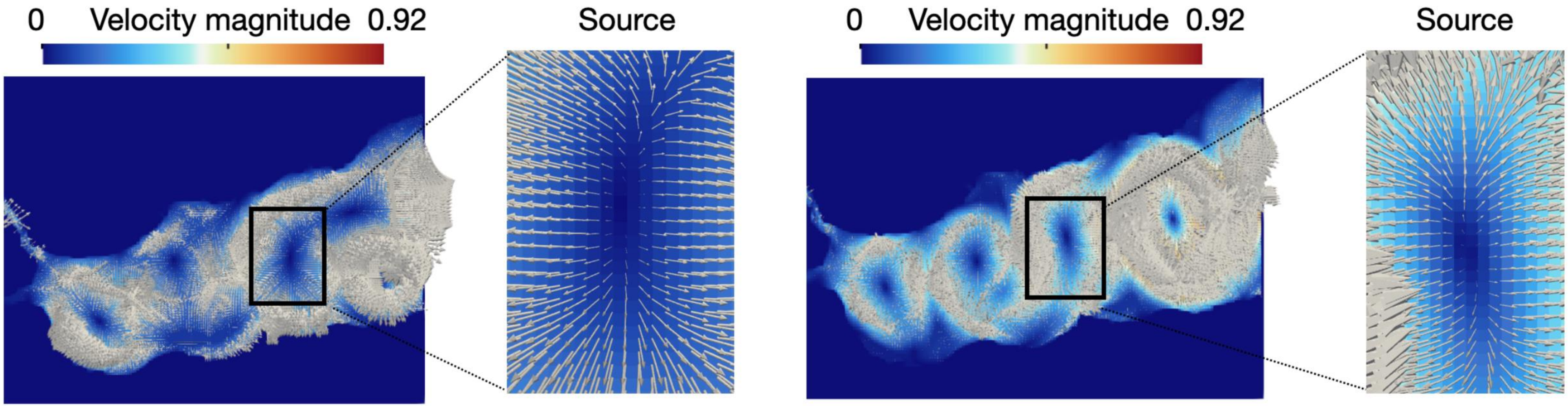
Analytical (our method): 0.0061 s
MC sampling (classical): 11.87 s

Speedup: 1945X

15 ensemble members

Red Sea Dataset

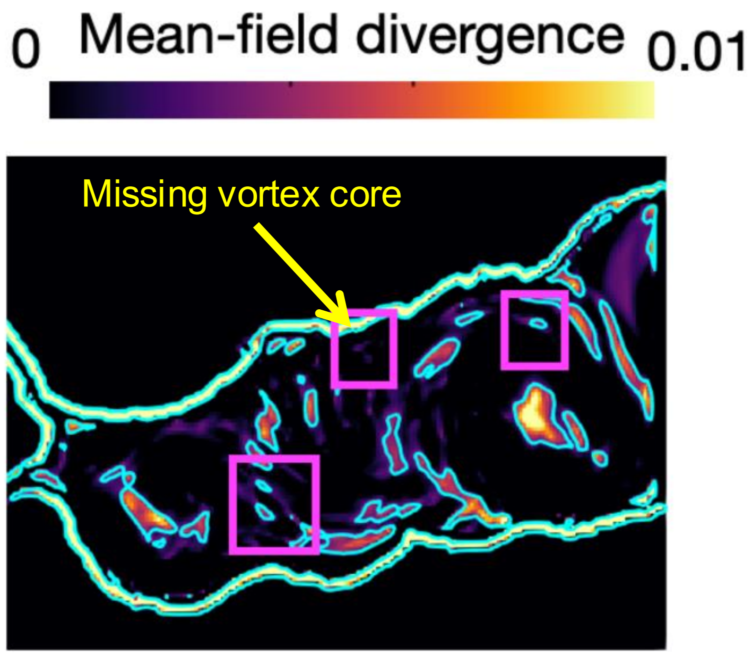
20 ensemble members

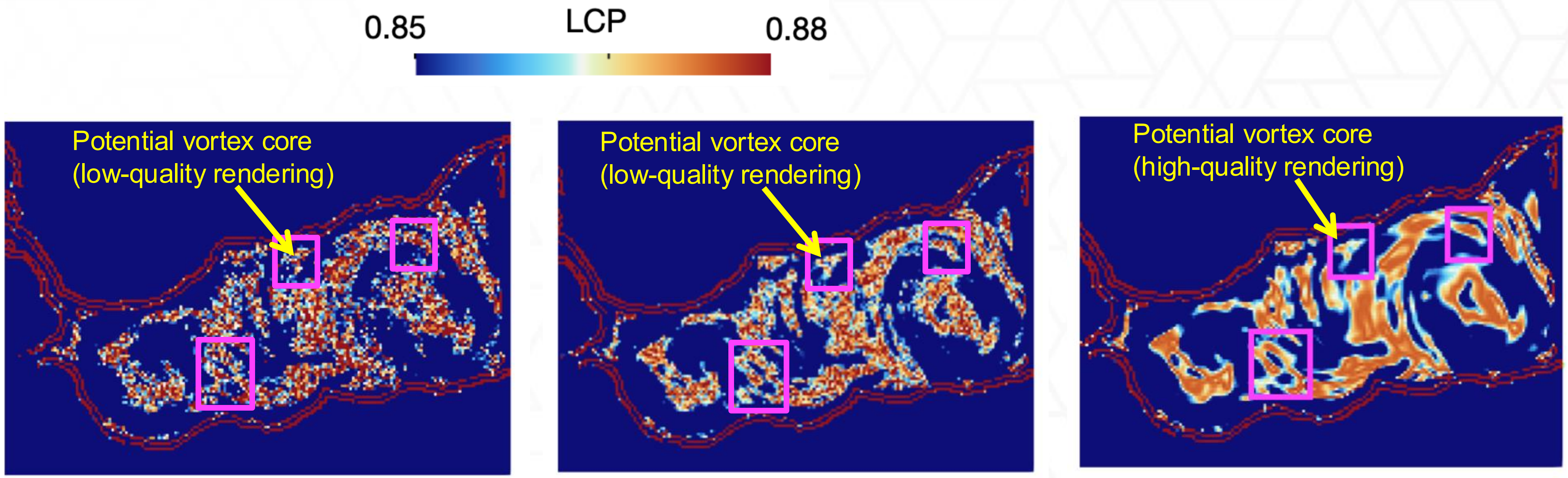


(a) Ensemble member 1

(b) Ensemble member 2

Red Sea Dataset





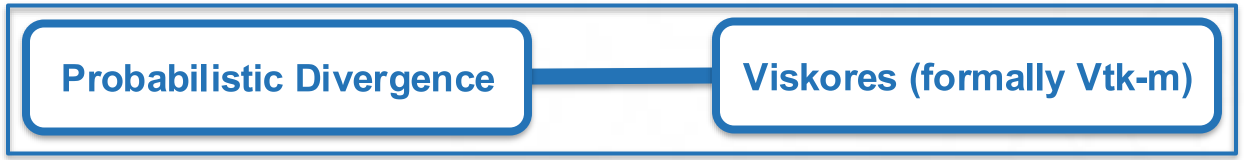
(a) Mean-field isocontour (cyan)

(b) MC probabilistic isocontour (500 samples)
Compute time = 315.174 s

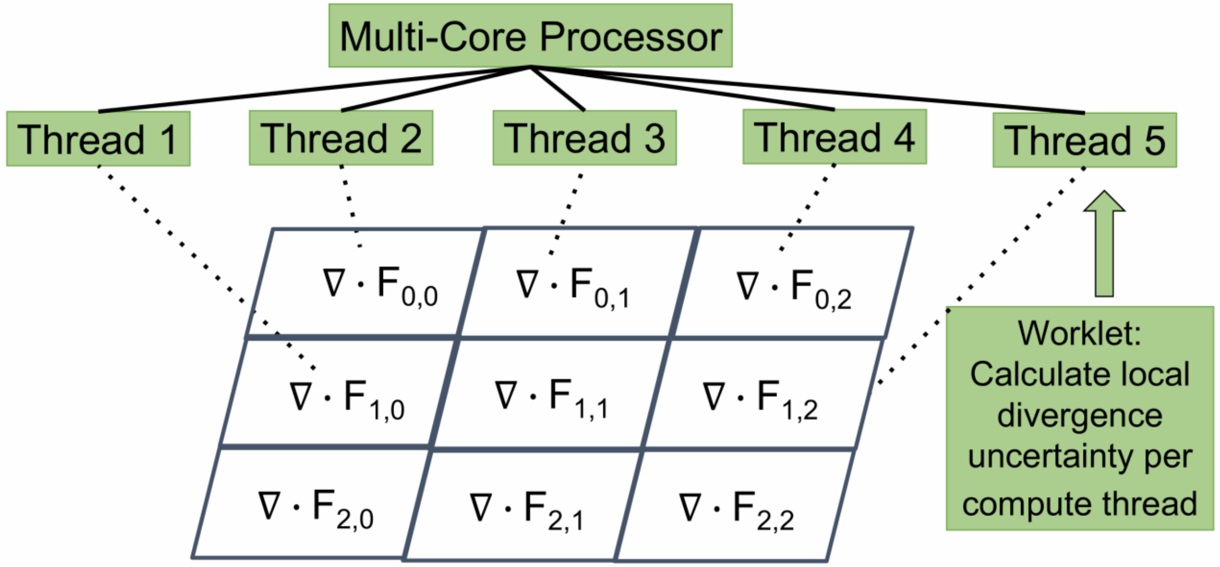
(c) MC probabilistic isocontour (2000 samples)
Compute time = 1469.58 s

(c) Our Analytical probabilistic isocontour
Compute time = 0.28 s
Speedup: 5248X

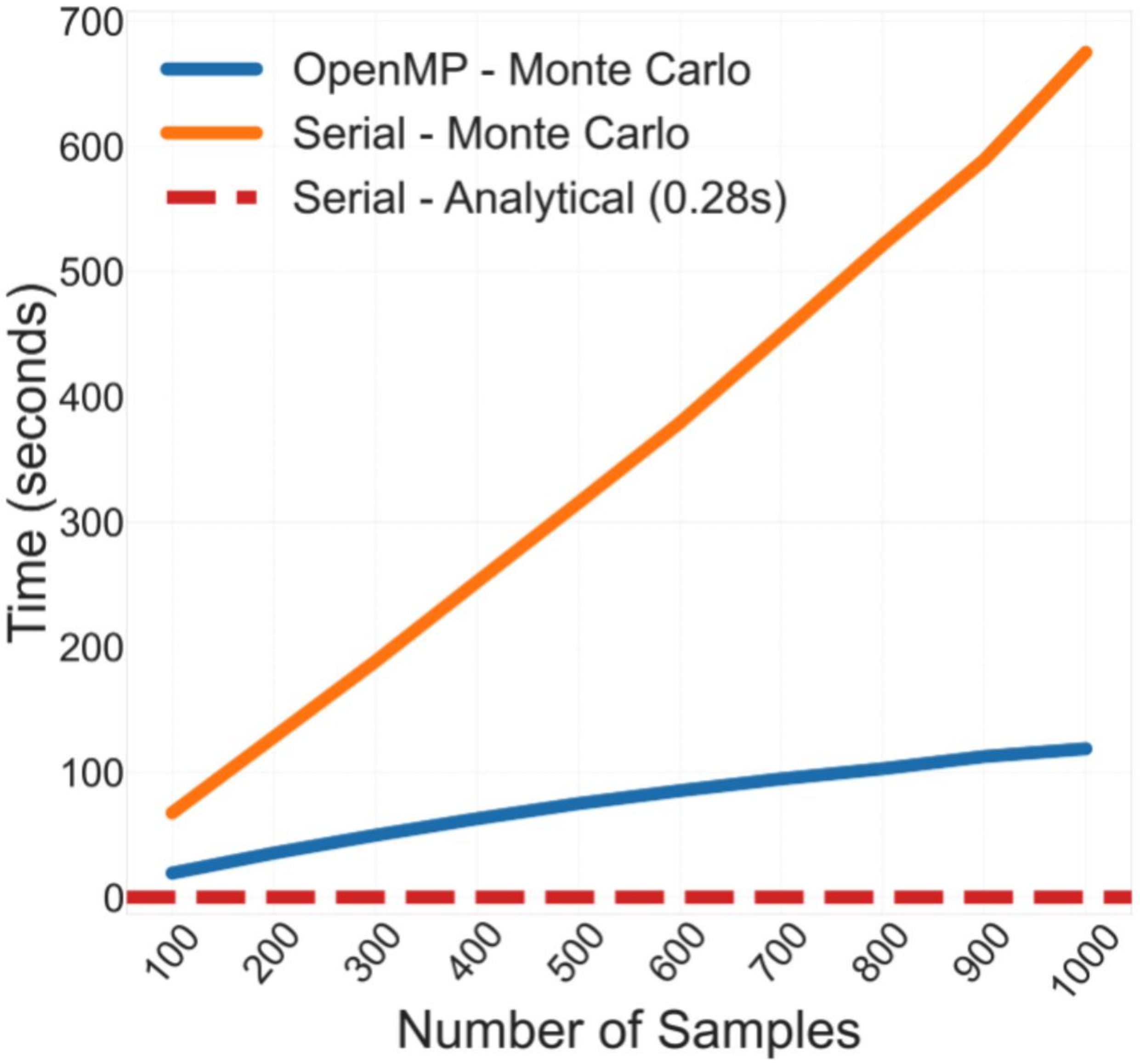
Performance Optimization and Evaluation



K. Moreland et al., “Vtk-m: Accelerating the visualization toolkit for massively threaded architectures”. IEEE Computer Graphics and Applications, 36(3):48–58, 2016



- Leverage parallelism across cell LCP computation
- **2X** Speedup AMD-GPU over OpenMP
- **1946X** Speedup analytical serial over MC serial
- **19698X** Speedup parallel analytical over MC serial



Remarks and Conclusion

Efficient visualization of local probabilistic divergence

Leverage Viskores (formally vtk-m) for performance portable optimization

Future Work

Extension to 3D and integrate to ParView

Correlation between vector components, neighbors, and divergence

Non-Gaussian distributions

Acknowledgements

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