Statistical Analysis for Uncertainty Quantification and Visualization of Scientific Data

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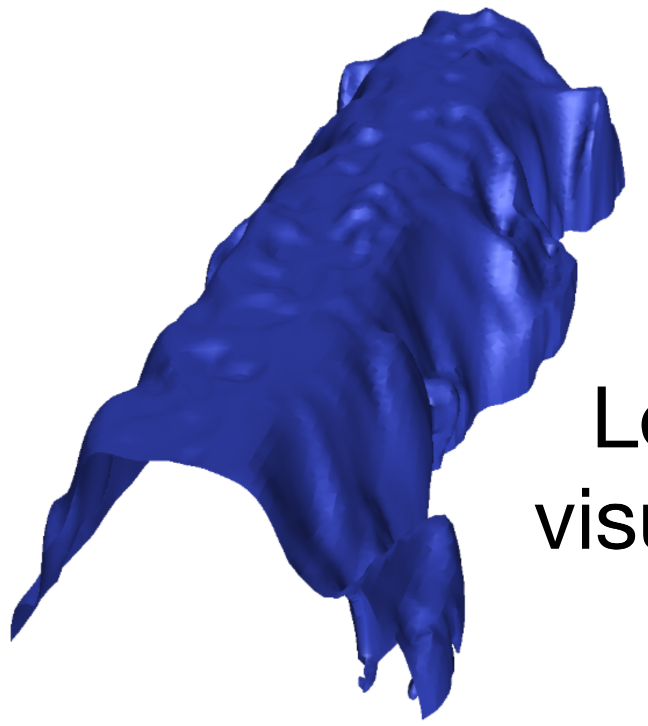
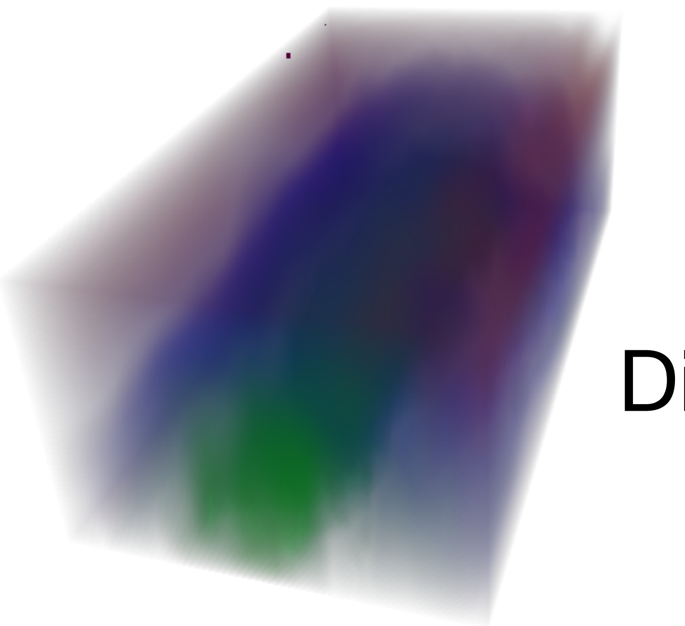
Outline

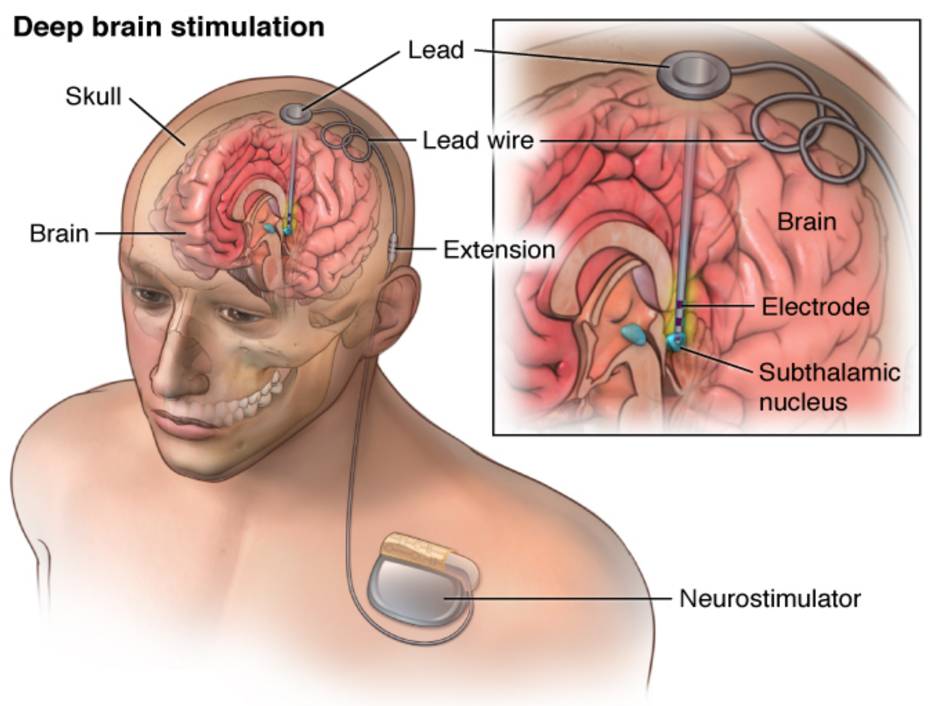
- Need for uncertainty visualization
- Uncertainty visualization for level sets
  - Marching squares/cubes algorithm in uncertain data, e.g., ensemble simulations
- Other applications of uncertainty visualization
  - Morse complex visualizations for ensembles
  - Direct volume rendering using ray casting
  - Deep brain stimulation (DBS) imaging
  - Electrodcardiographic Imaging (ECGI)
- Conclusion and future work



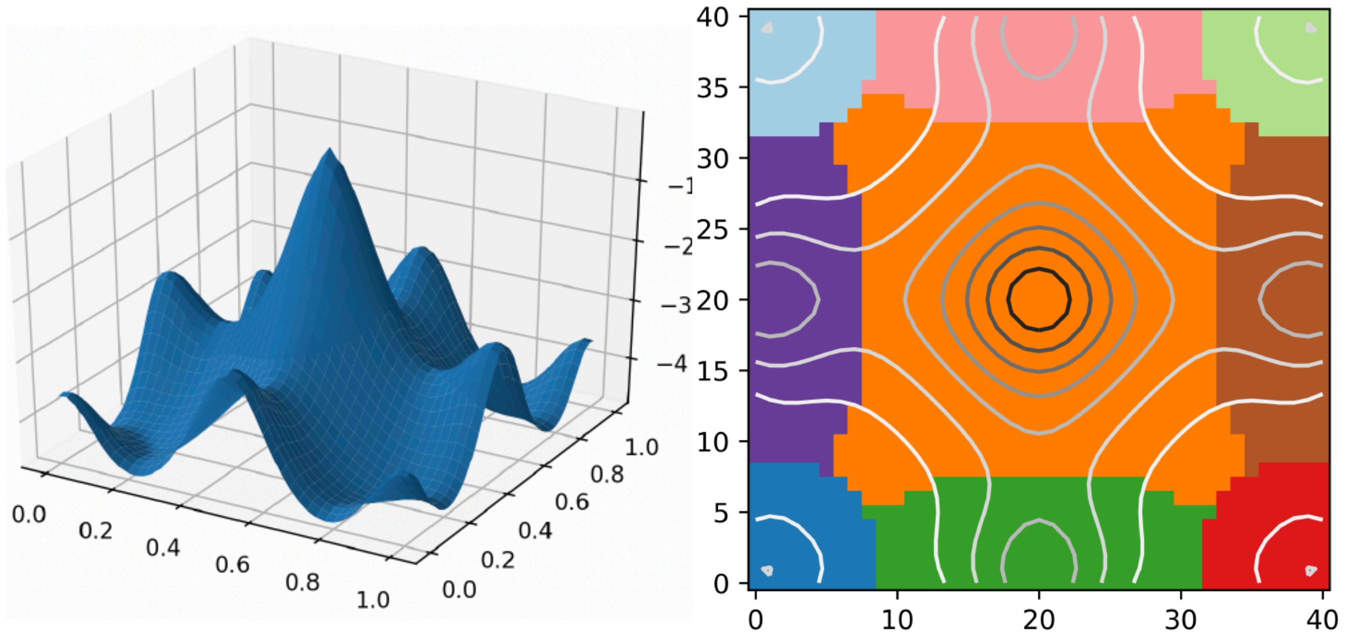
Level-set visualization



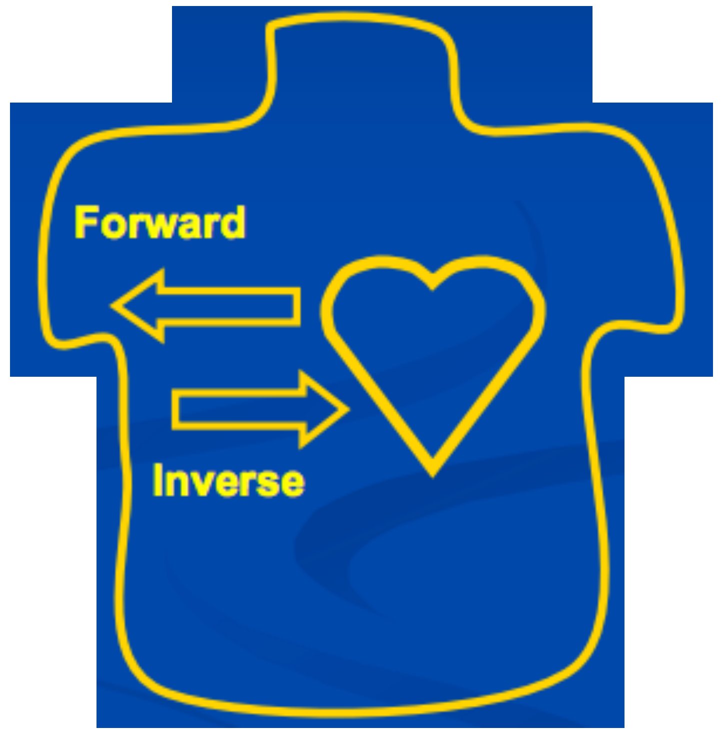
Direct volume rendering



DBS



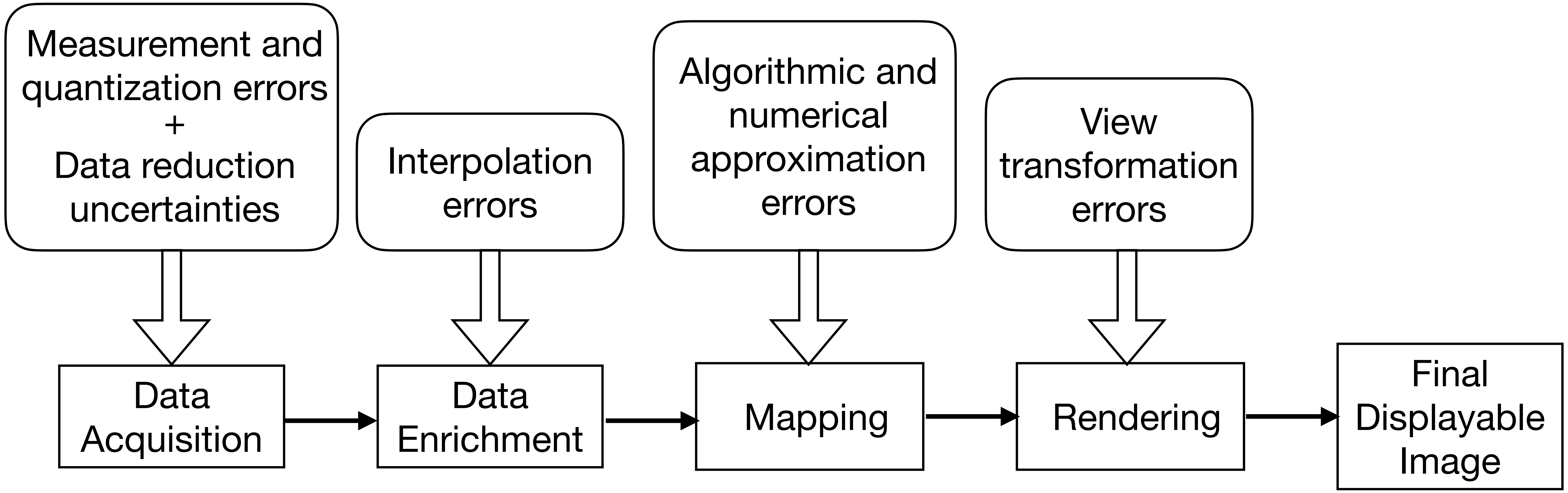
Morse complex visualization



ECGI

Why Visualize Uncertainty? [Johnson and Sanderson, 2004]

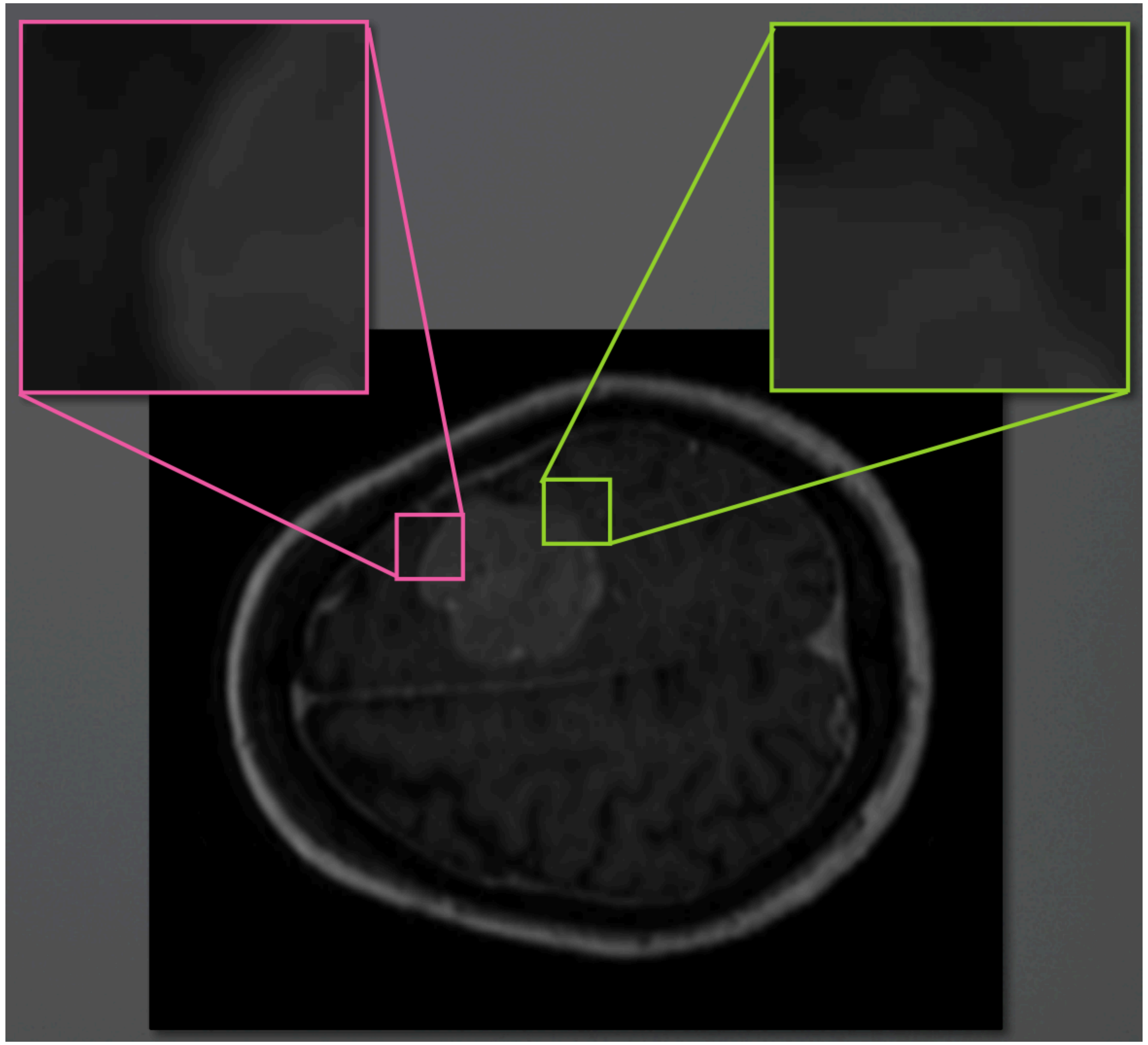
Minimize risks associated with scientific decisions



The Visualization Pipeline
[Brodlie et al., 2012]

Why Visualize Uncertainty?

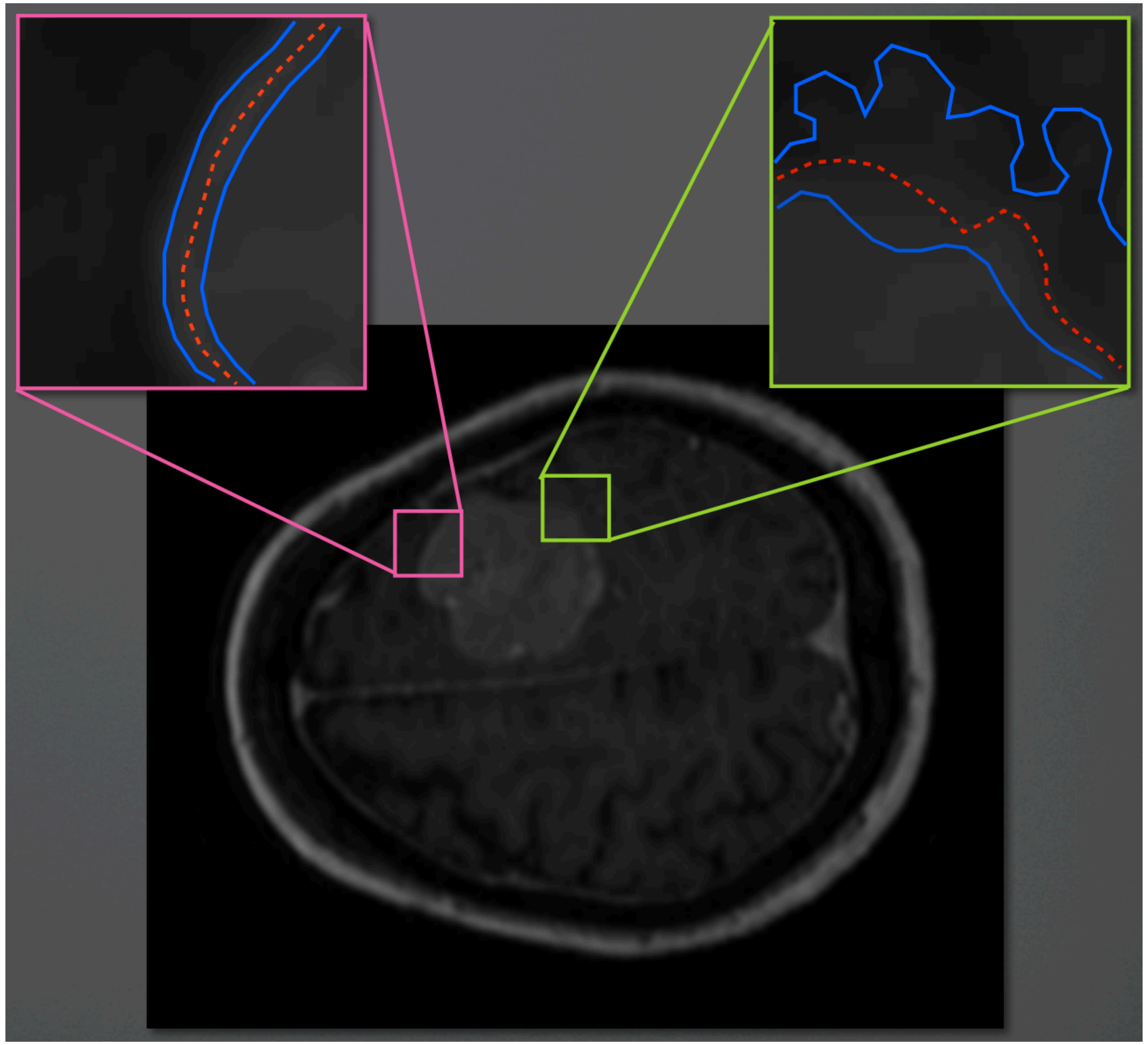
Can you identify a tumor boundary?



Whole brain atlas: http://www.med.harvard.edu/AANLIB/home.html

Why Visualize Uncertainty?

Can you identify a tumor boundary?

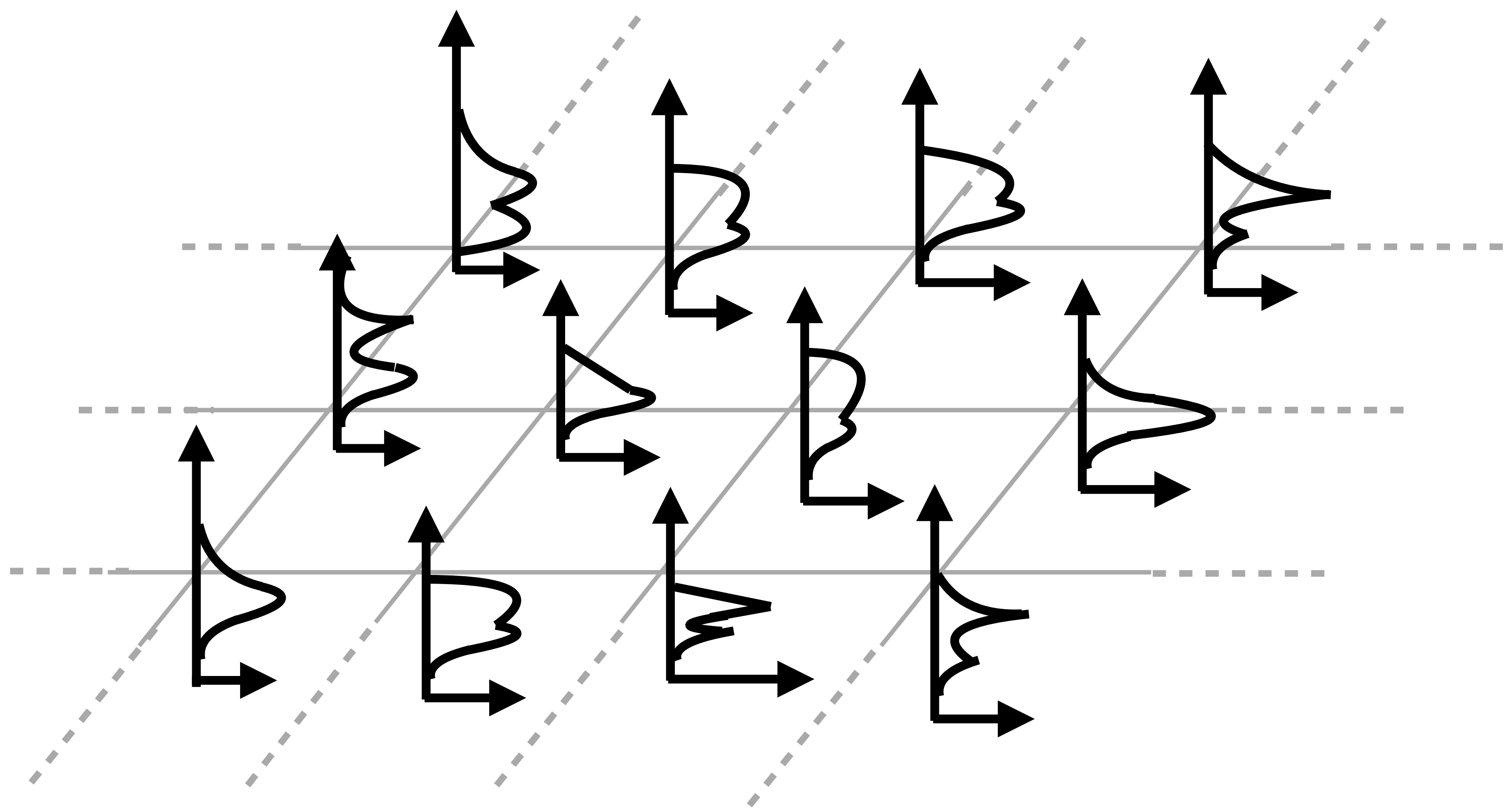


Whole brain atlas: http://www.med.harvard.edu/AANLIB/home.html

Data Reduction and Distributions

Distributions: Hixel representation/ in-situ statistical summaries for large-scale data [Thompson et al., 2011, Lehmann and Jung, 2014, Hazarika et al., 2018]

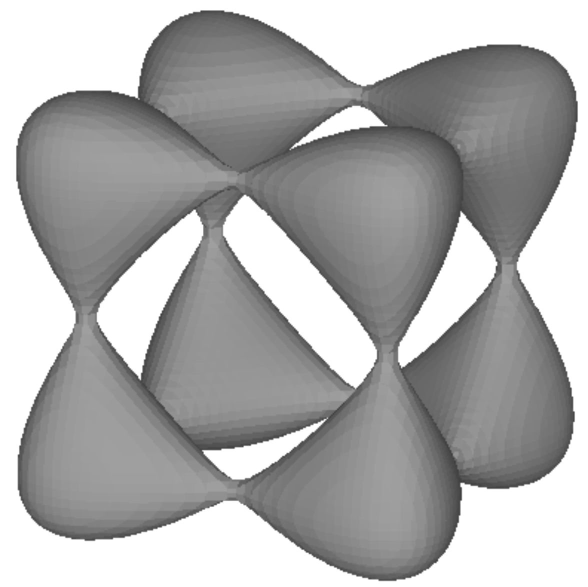
Ensemble Data: Multiple simulations for PDE solutions (Store min/max, Approximate distributions from samples)



Hixel representation: Storing a histogram/probability distribution at each vertex of a scalar grid

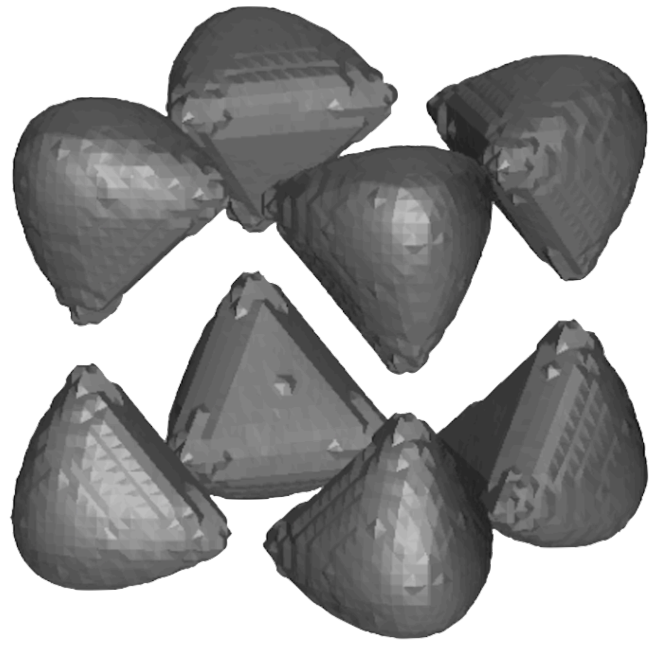
Data Reduction and Distributions

[Athawale et al., 2016]



Ground truth

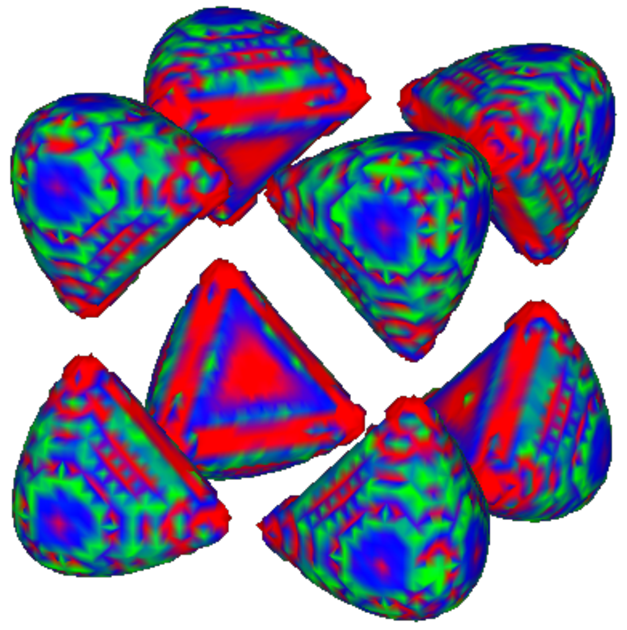
Memory consumption = 100*X



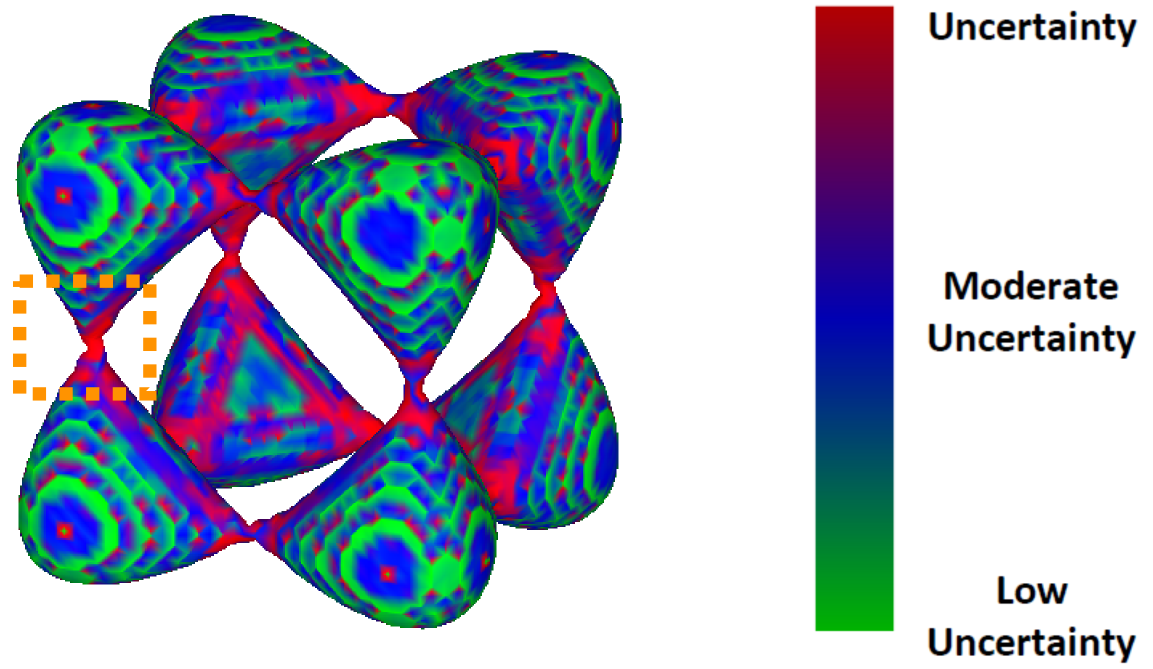
Mean

Memory consumption = X

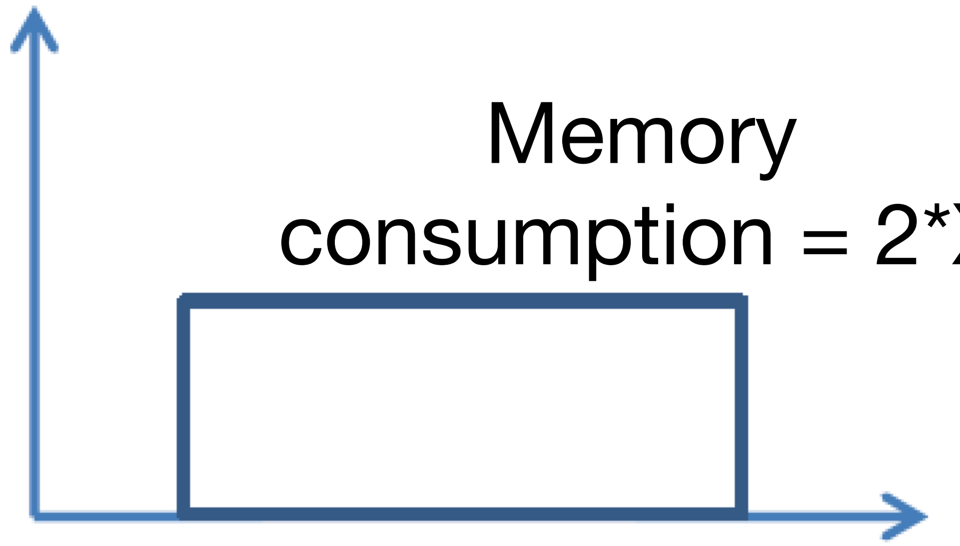
Brick size = 100



Parametric

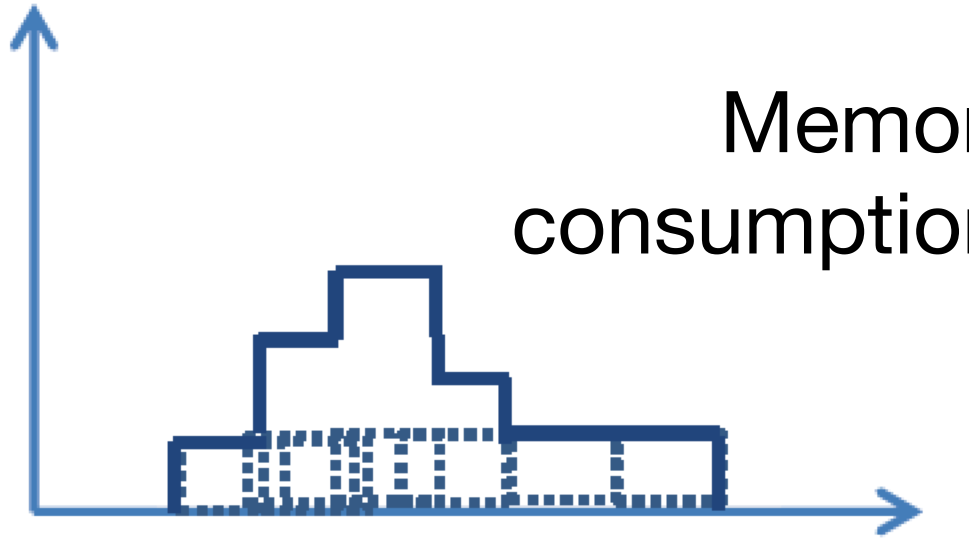


Nonparametric



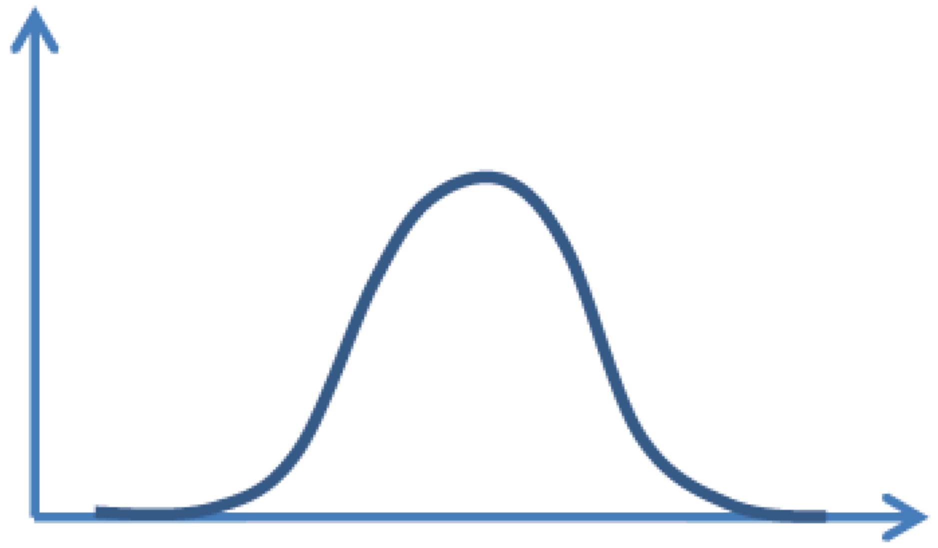
Memory consumption = 2*X

Uniform

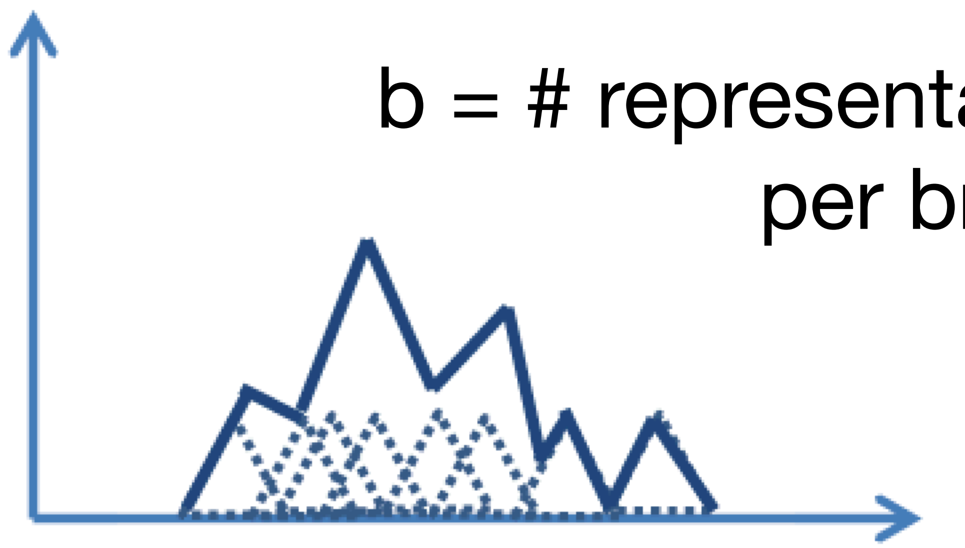


Memory consumption = b*X

Uniform kernel



Gaussian

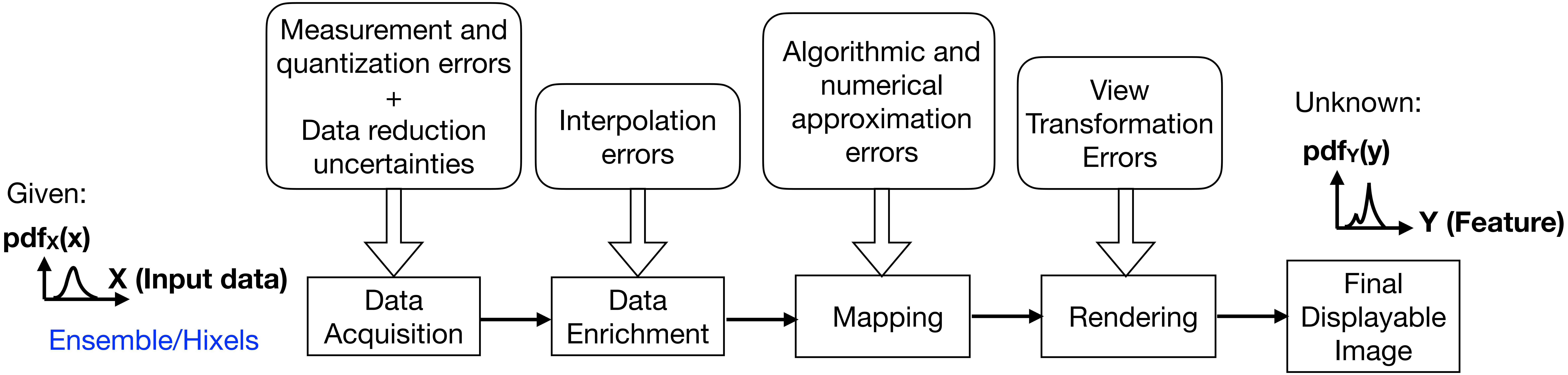


b = # representative samples per brick

Triangular kernel

Uncertainty Quantification (Abstract Statistical Approach)

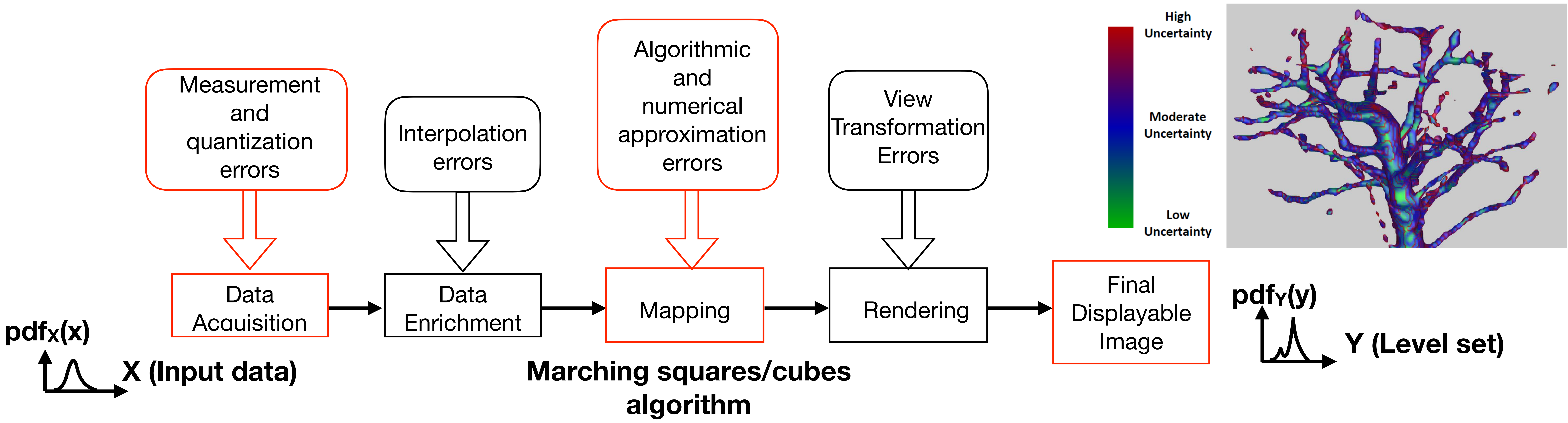
Monte Carlo (easy but expensive) vs. Analytical (difficult but fast)



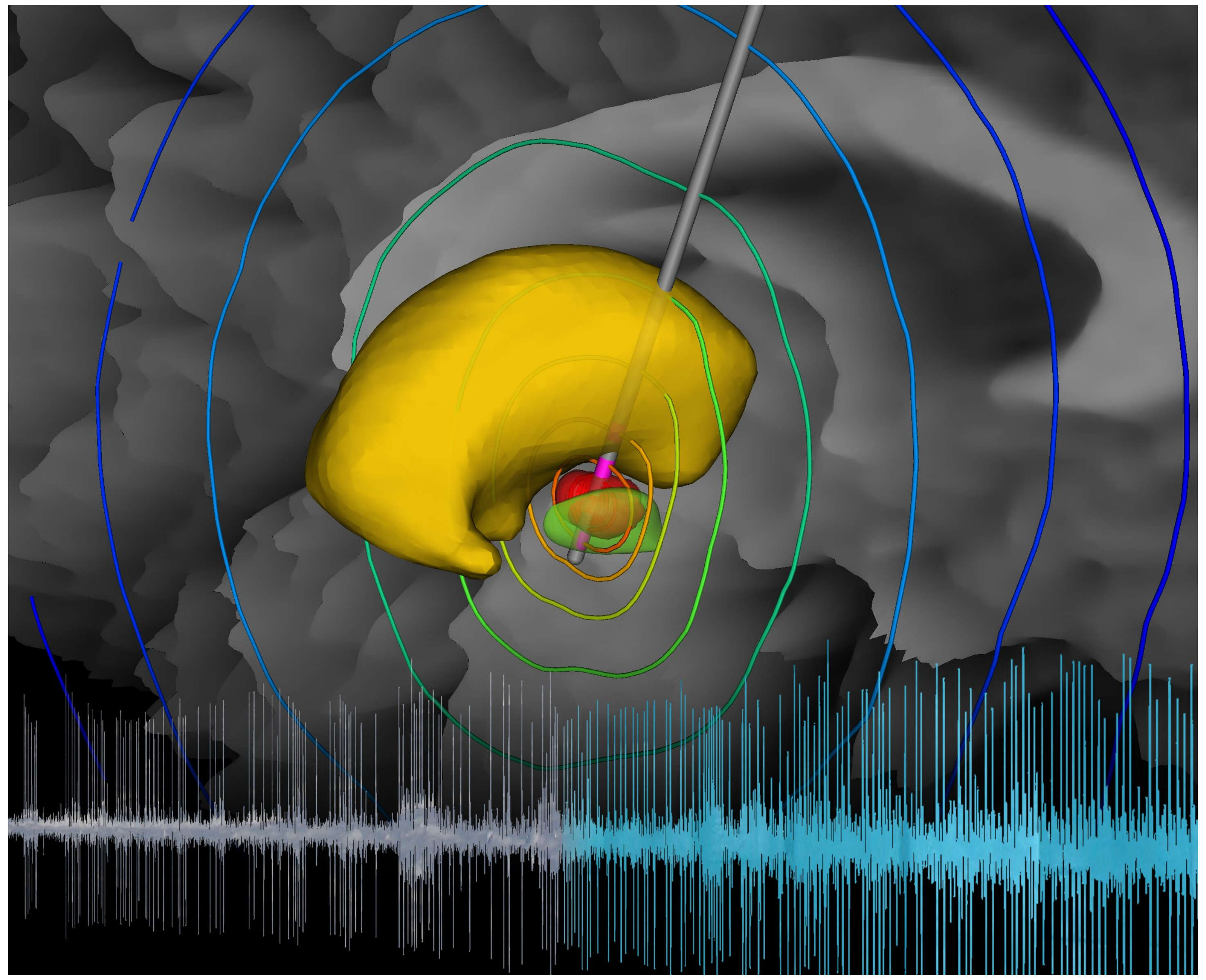
The Visualization Pipeline

Level-Set Extraction in Uncertain Data

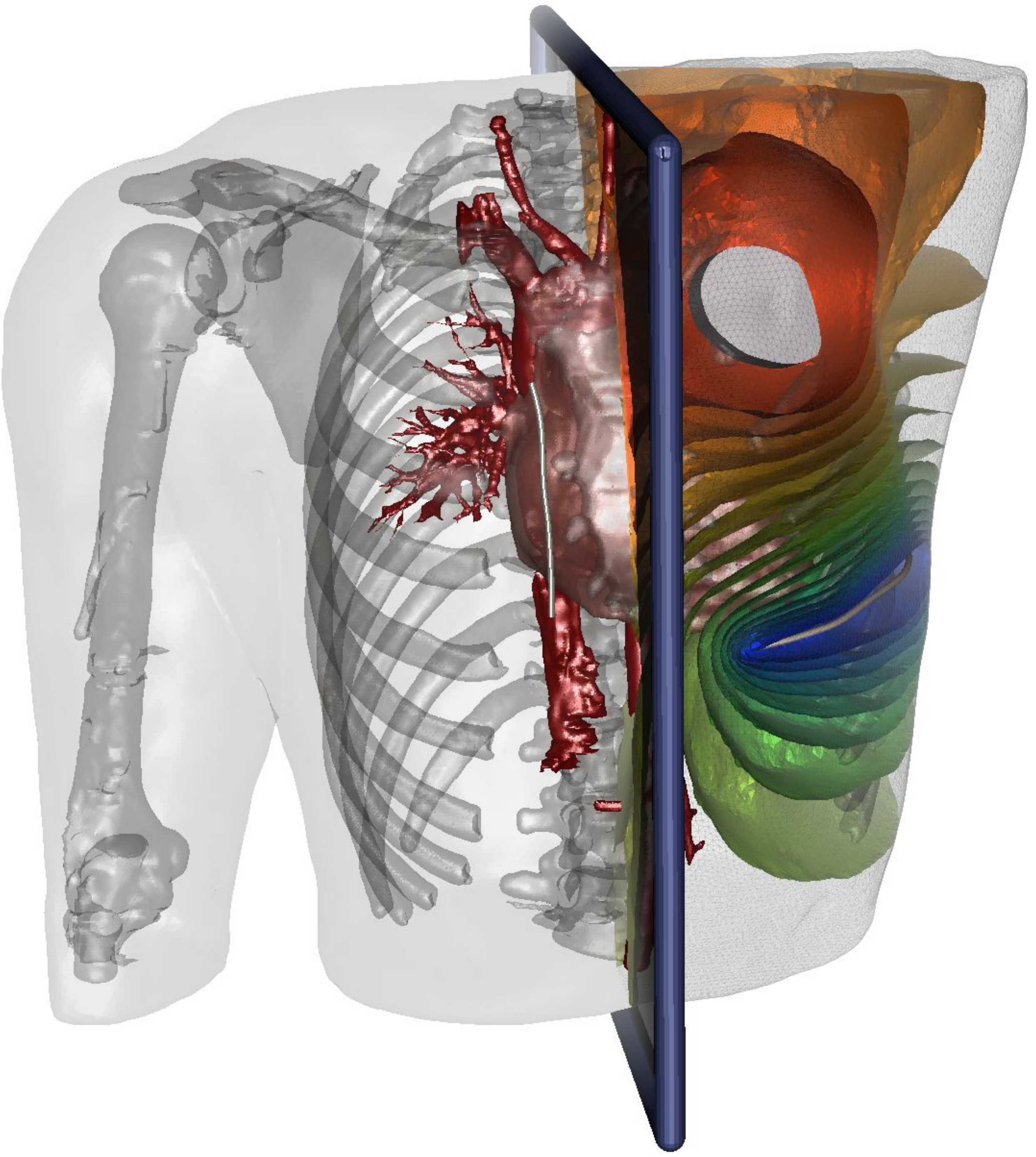
- Level sets and data uncertainty
- Marching squares/cubes algorithm in certain vs. uncertain data (our contribution!)
- Results



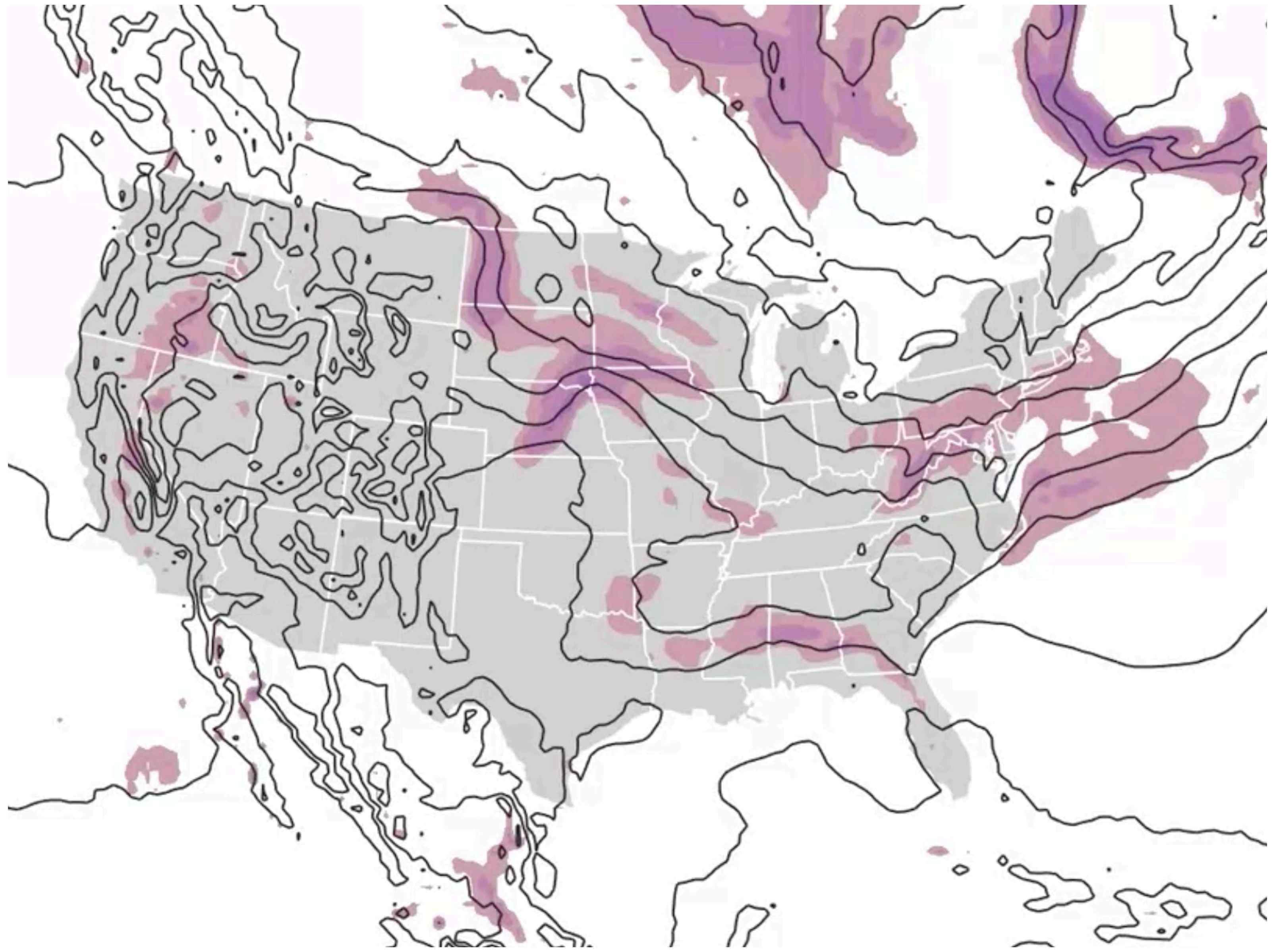
Level-Set Visualization



Deep Brain Stimulation (DBS)



Bioelectric-field Simulation

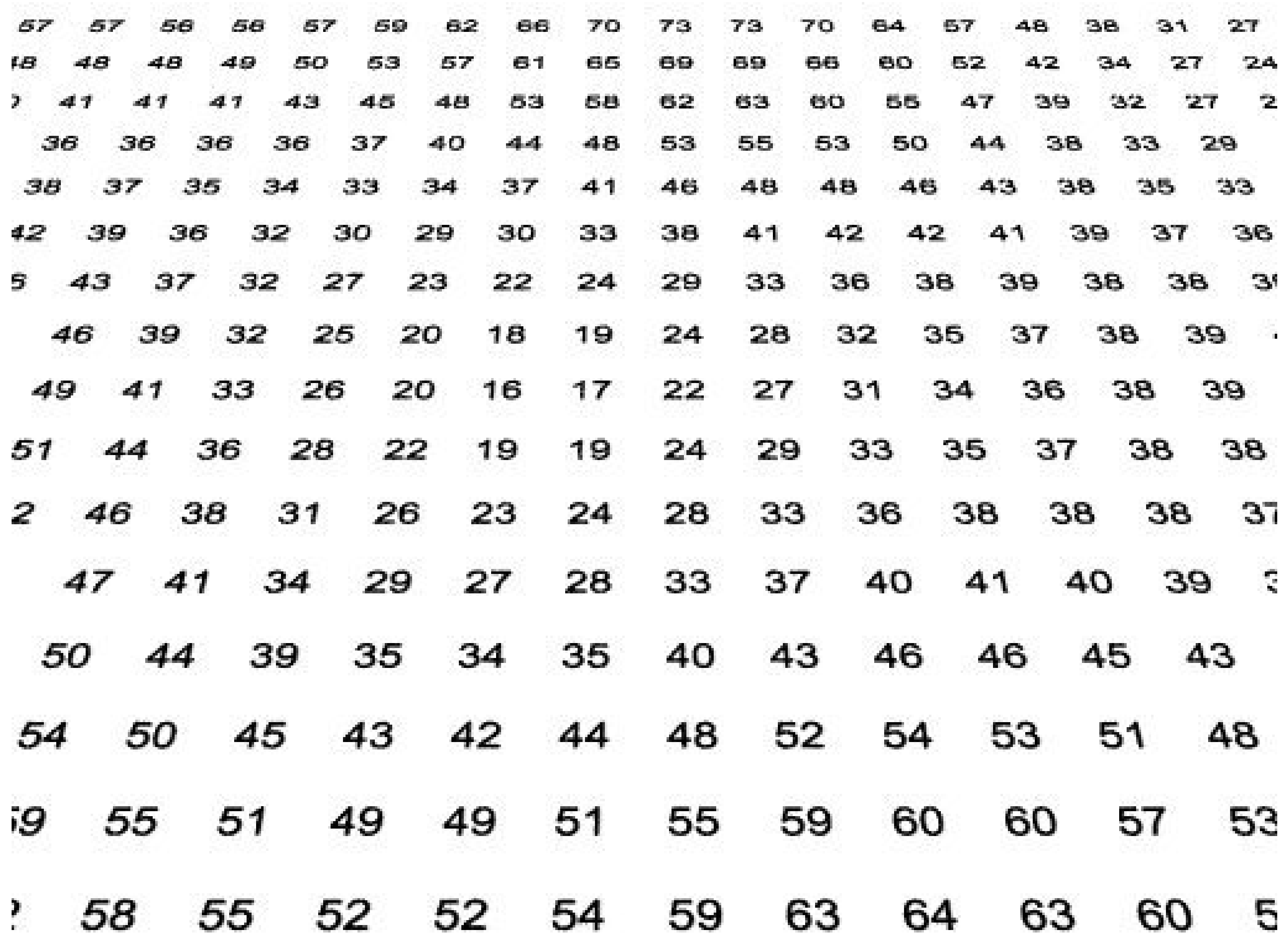


Temperature Field

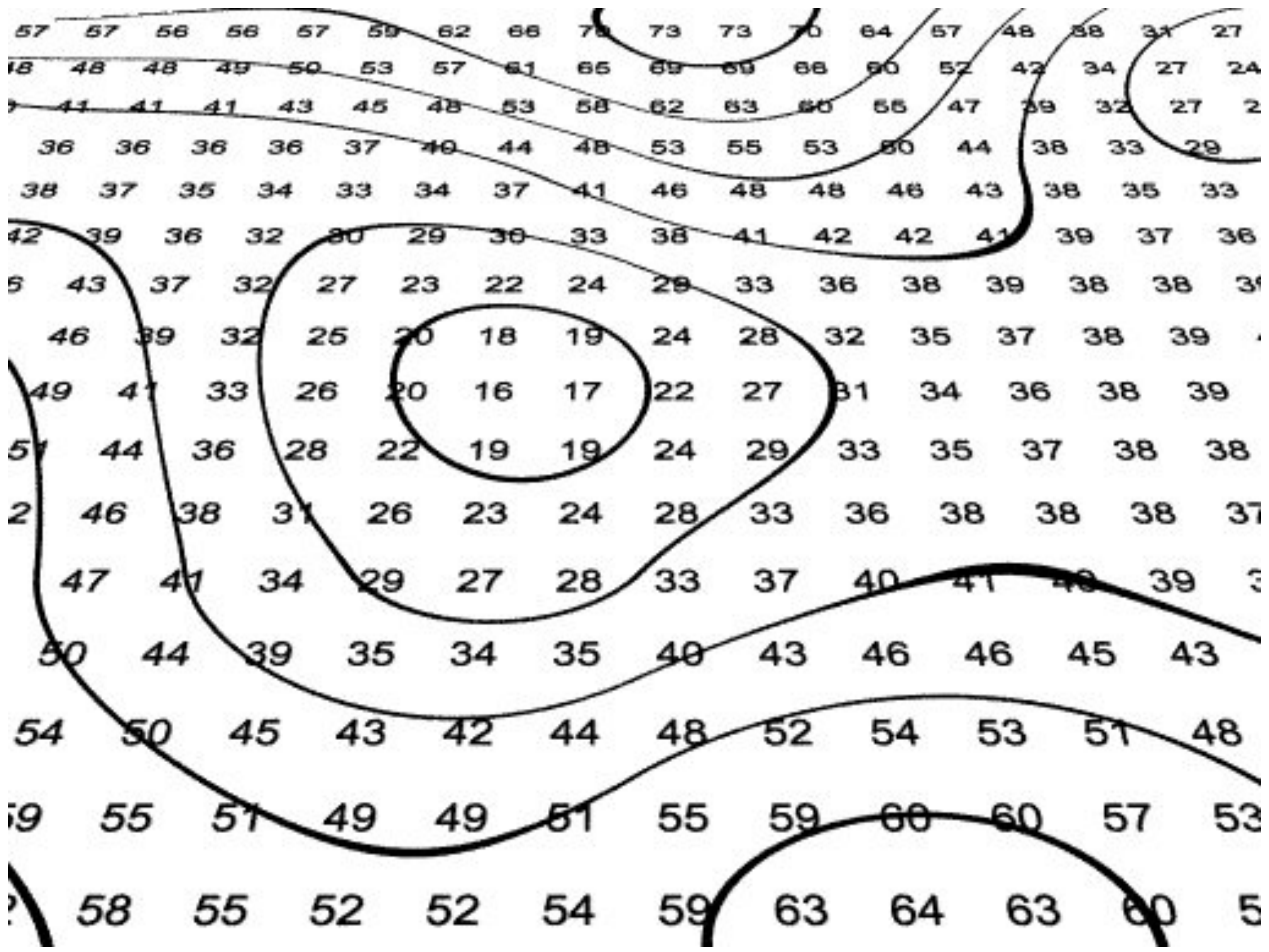
Level-Set Extraction

The inverse problem: The level-set *S* corresponding to the isovalue *k* is given by:

$$S = \{x \in \mathbb{R}^n | f(x) = k\} \text{ , where } \quad f : \mathbb{R}^n \to \mathbb{R}$$



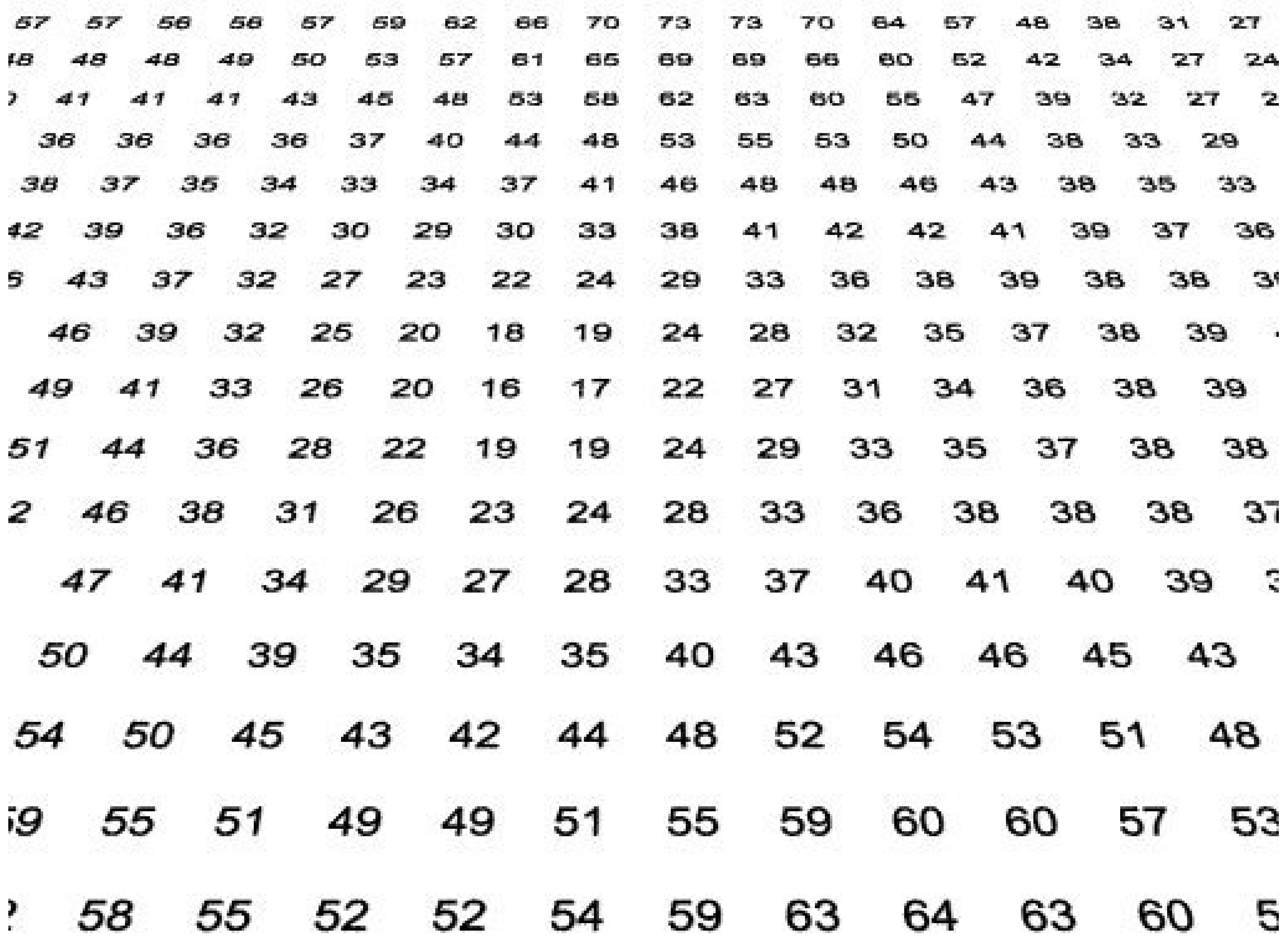
Input: Scalar Field



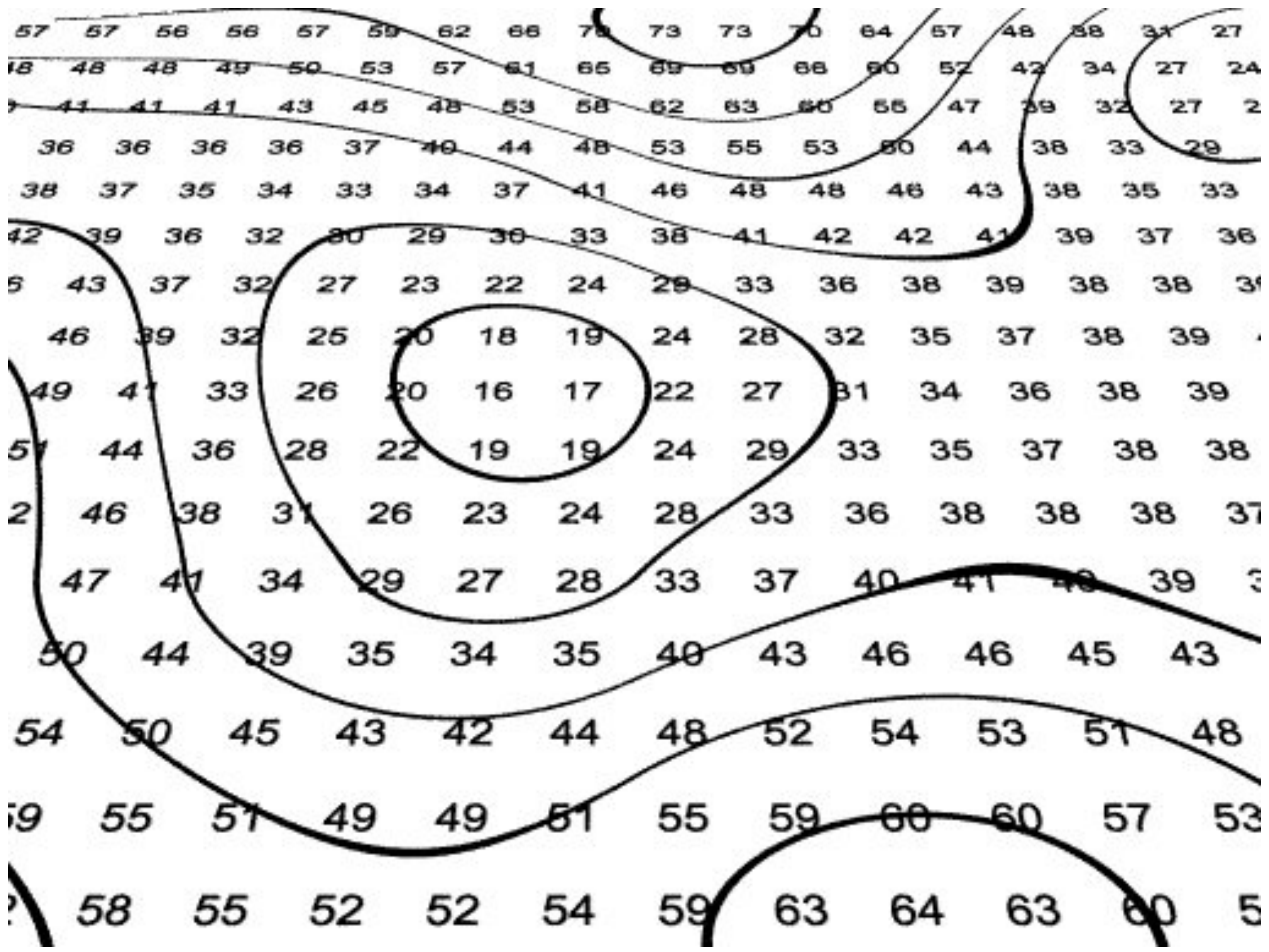
Output: Level-Sets Visualization

Level-Set Extraction in Uncertain Data

Research Question: Analysis of topological and geometric variations in level sets for uncertain scalar field



Input: Noisy Scalar Field

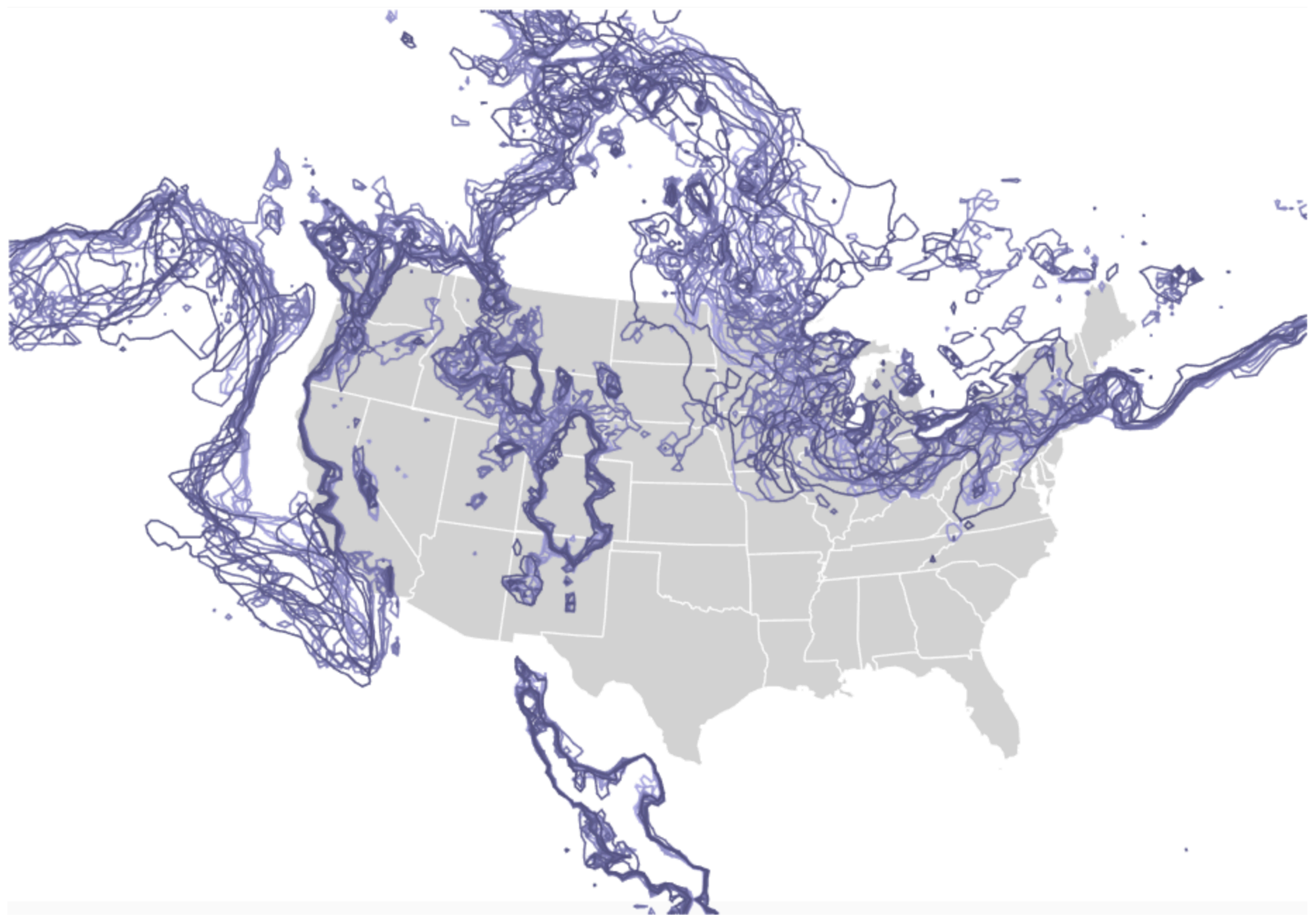


Output: Level-Sets Visualization

May not represent the true level sets!

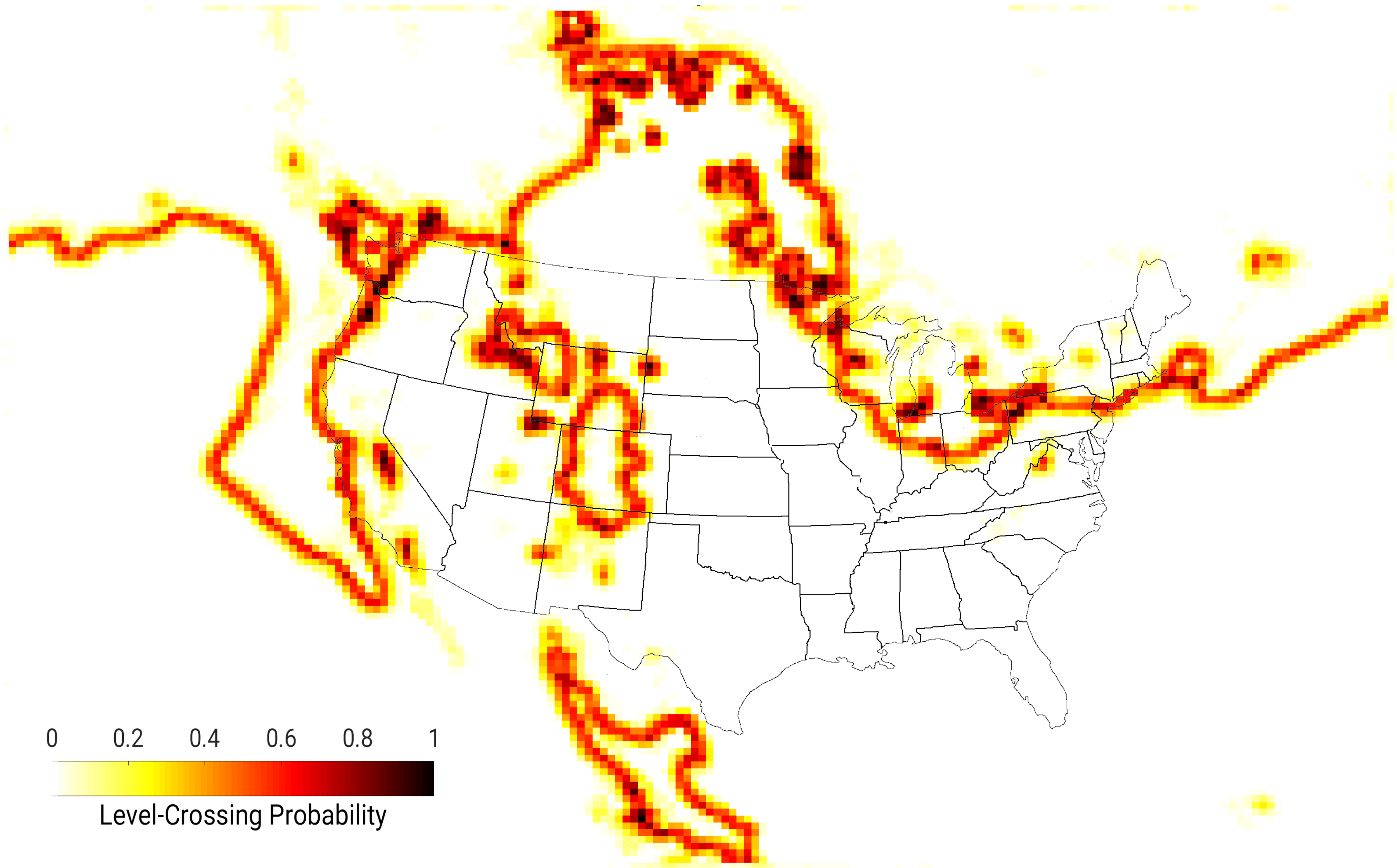
Visualization of Level-Sets in Uncertain Data

Spaghetti plots
[Potter et al., 2009]

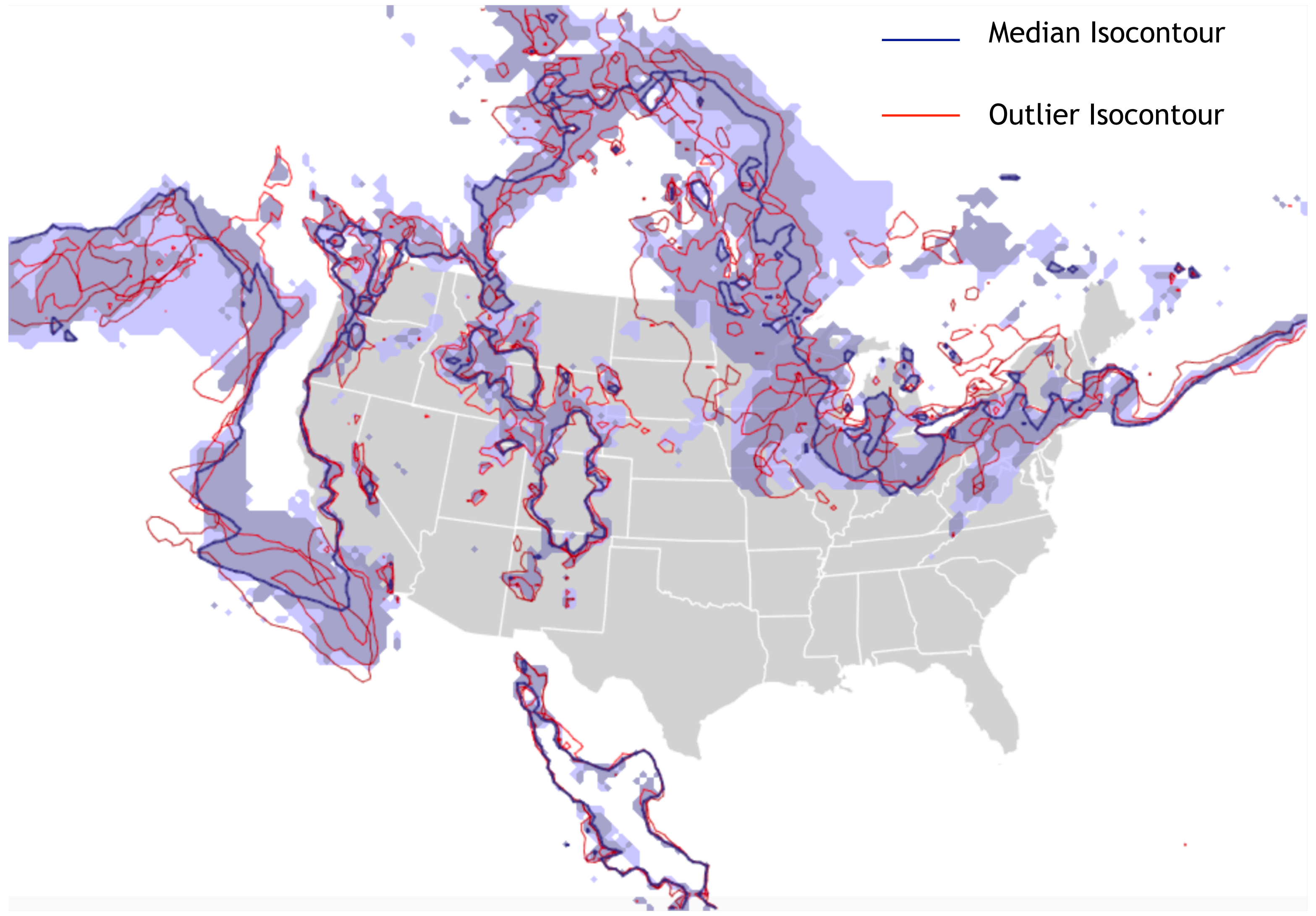


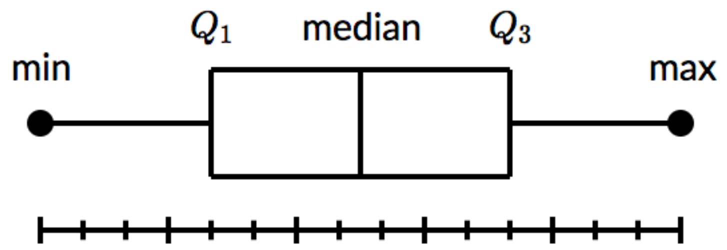
Visualization software: The WeaVER
[Quinan and Meyer, 2016]

Probabilistic marching cubes
[Pöthkow et al., 2011]



Contour/Surface box plots
[Whitaker et al., 2013; Genton et al., 2014]





1D box plot

The visualization of uncertain temperature field
isovalue (k) = 60°F

www.sci.utah.edu

Marching Squares/Cubes Algorithm for Level-Set Extraction

[Lorenson and Cline, 1987]

Google Scholar

Bill Lorensen

GE Global Research (retired)
Verified email at nycap.rr.com - Homepage

FOLLOW

TITLE	CITED BY	YEAR
Marching cubes: A high resolution 3D surface construction algorithm WE Lorensen, HE Cline ACM siggraph computer graphics 21 (4), 163-169	15483	1987
Object-oriented modeling and design J Rumbaugh, M Blaha, W Premerlani, F Eddy, WE Lorensen Prentice-hall 199 (1)	11730	1991
The visualization toolkit: an object-oriented approach to 3D graphics WJ Schroeder, B Lorensen, K Martin Kitware	3994	2004

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Citations	36273	5984
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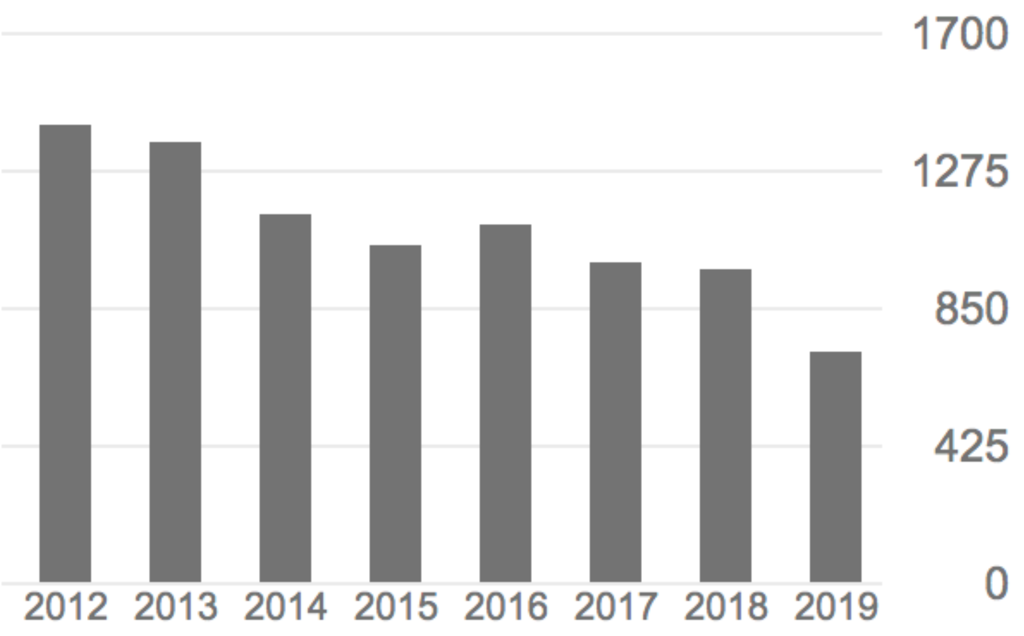
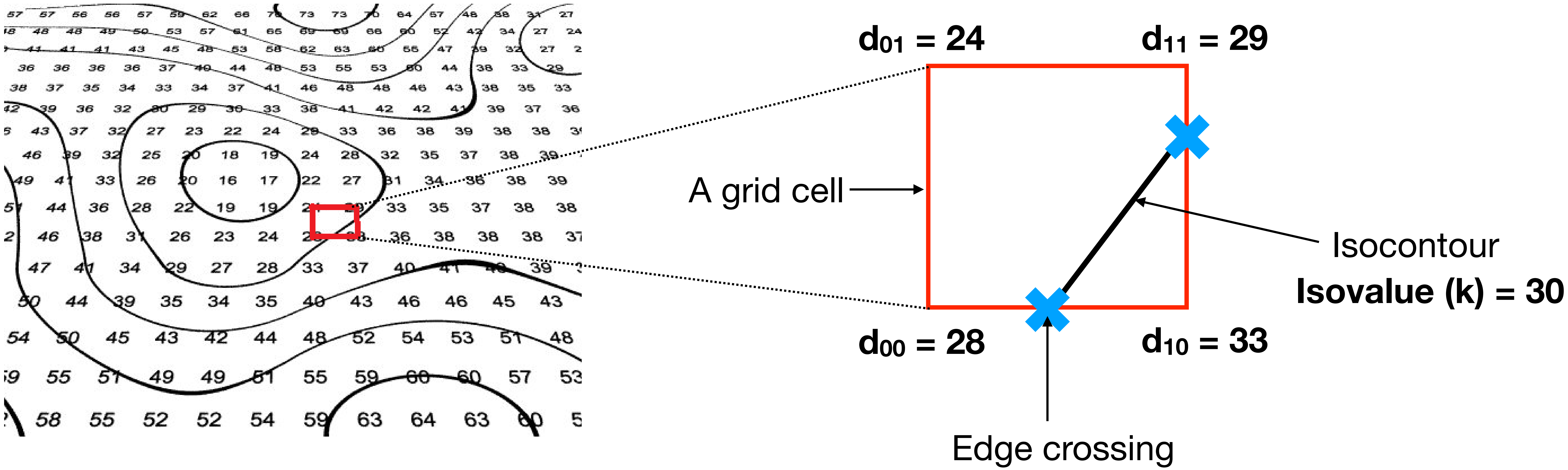


Marching Squares Algorithm (MSA)

- Bilinear interpolation: prediction of unknown data values within a grid cell
- For each cell:
  - Extract isocontour topology (Which cell edges are crossed?)
  - Compute geometry (Where on the cell edge?)



MSA: The Topology Step (which edges?)

Data (d_{xy}) > Isovalue (k) : Positive vertex (+)
Data (d_{xy}) < Isovalue (k) : Negative vertex (−)

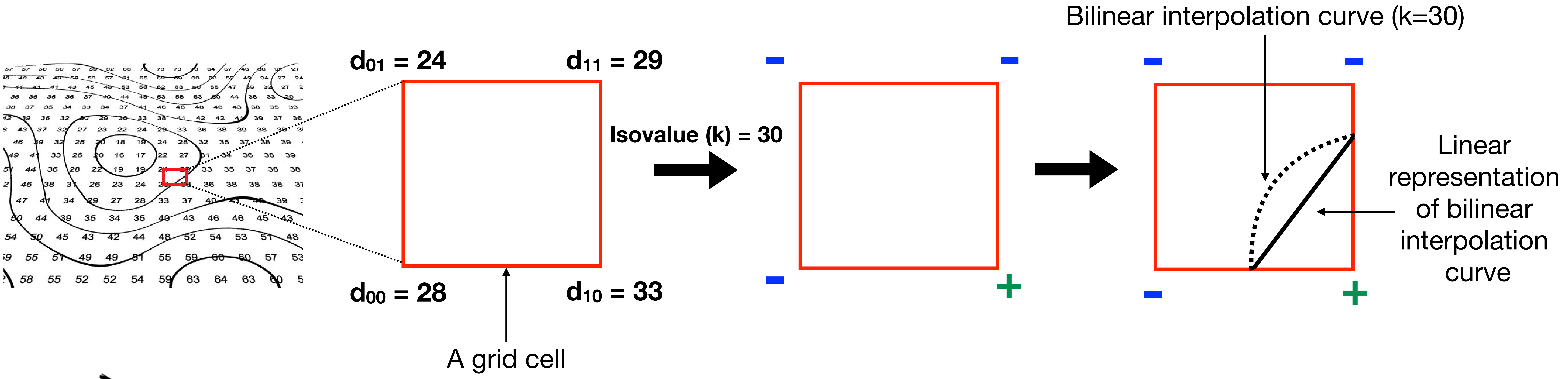
Bilinear interpolation function:

$f: [0,1] \times [0,1] \rightarrow R$

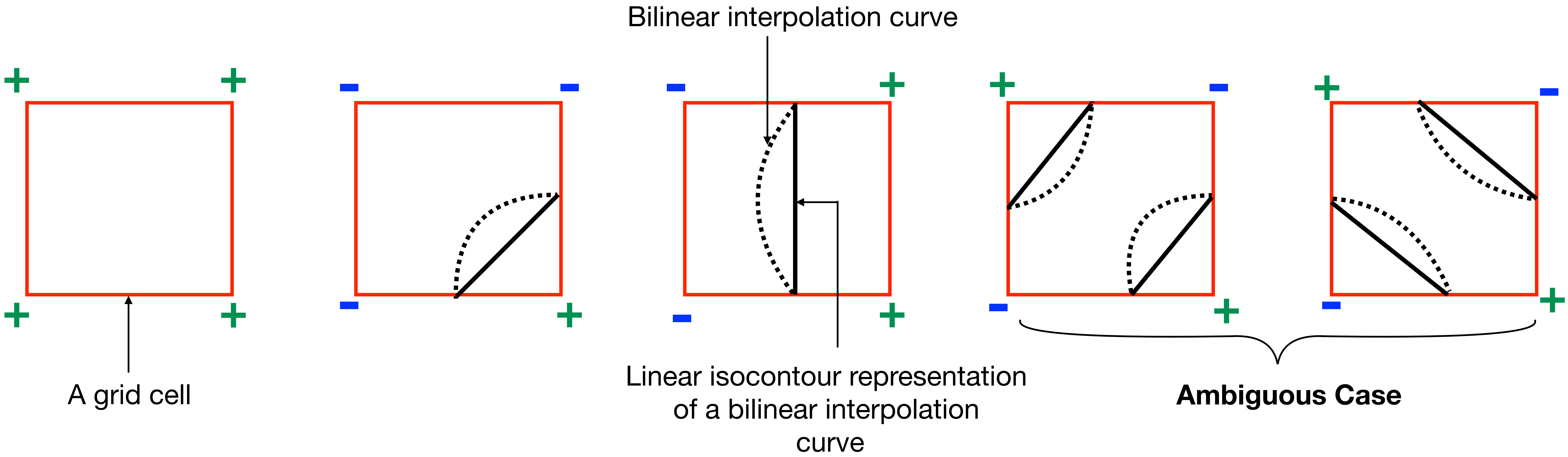
$f(x,y) = ax + by + cxy + d$ (the equation of hyperbola!)**, where**

$a = d_{10} - d_{00}$, $b = d_{01} - d_{00}$

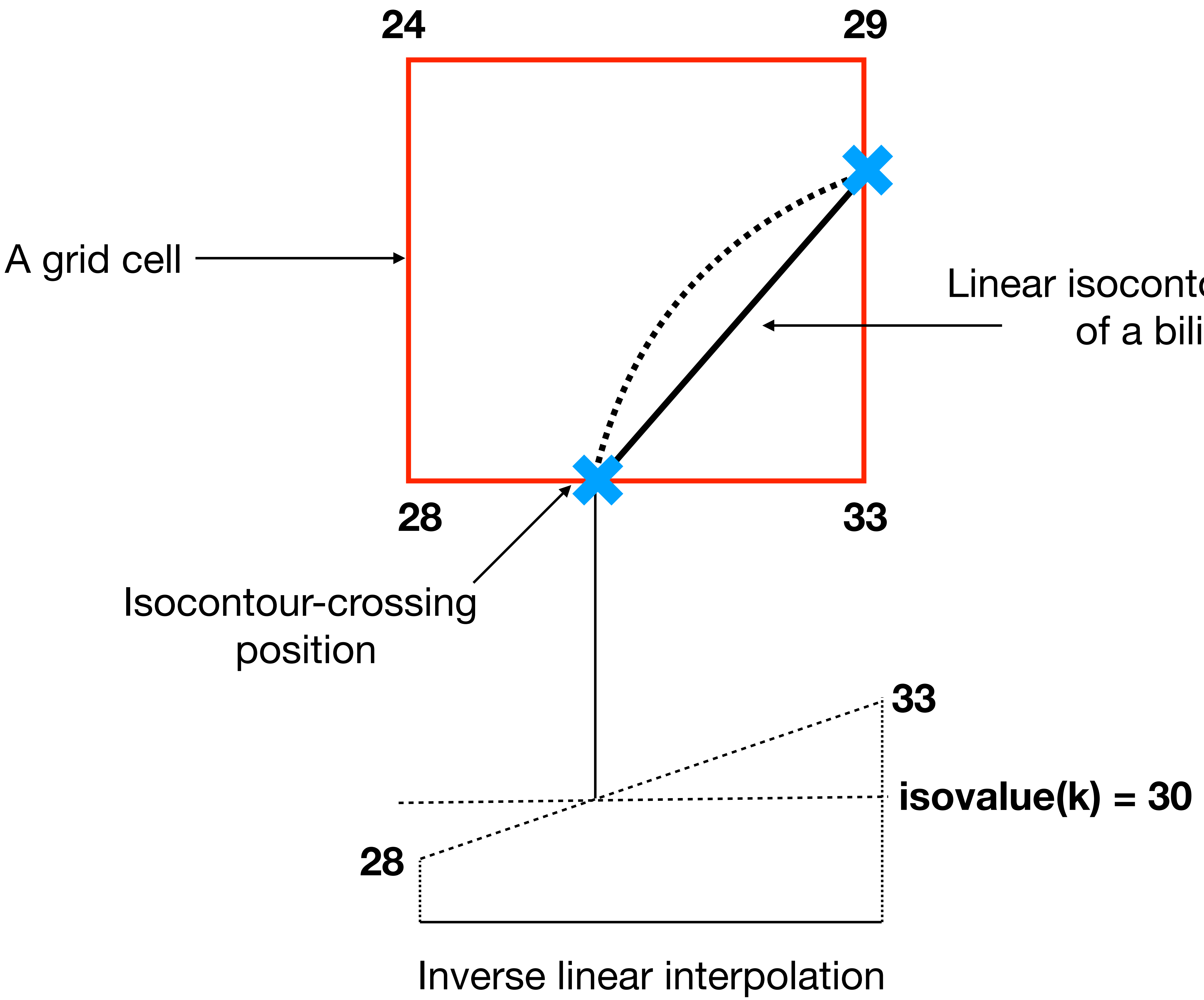
$c = d_{00} + d_{11} - d_{01} - d_{10}$, $d = d_{00}$



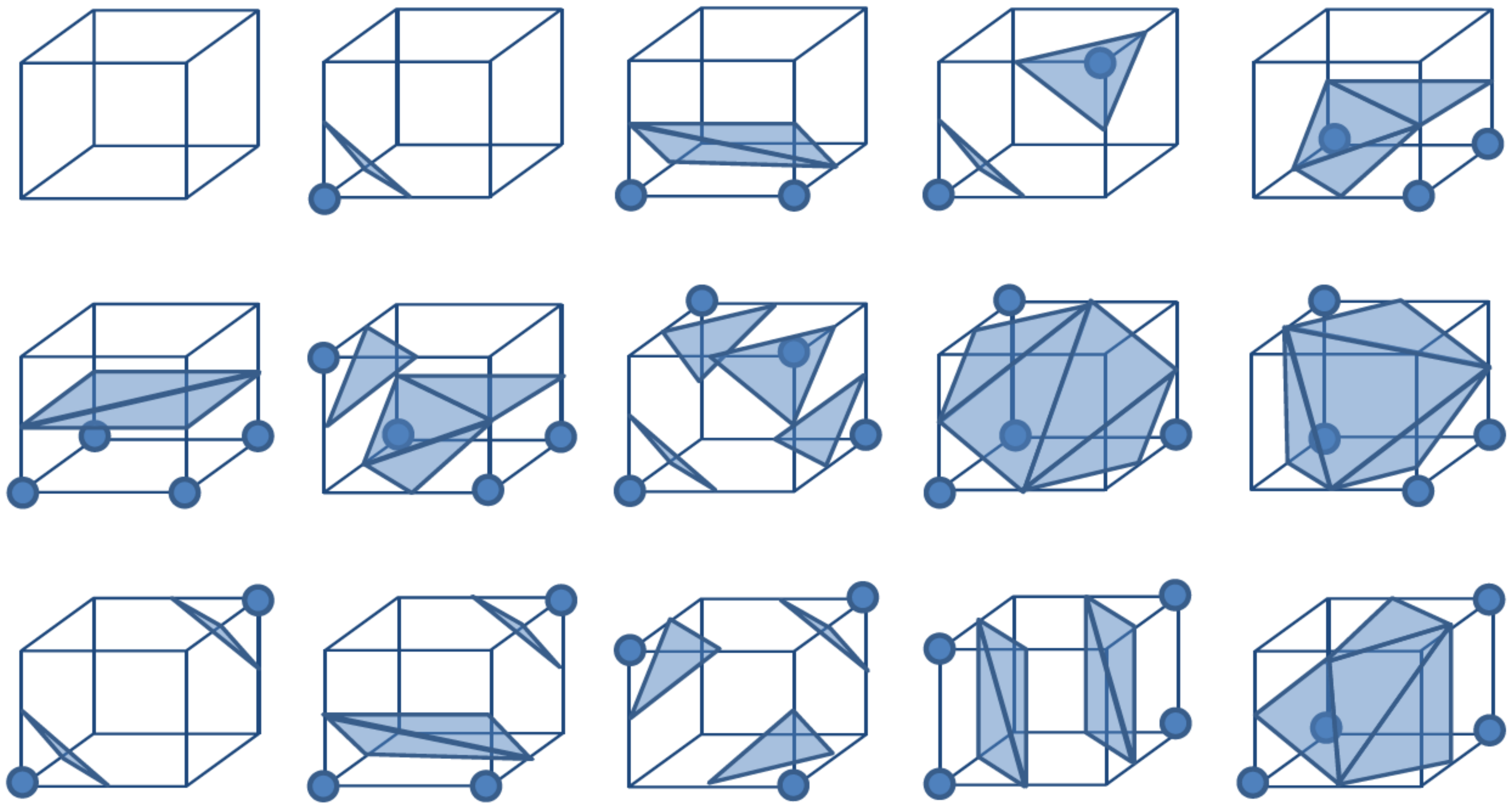
MSA: Topological Cases (which edges?)

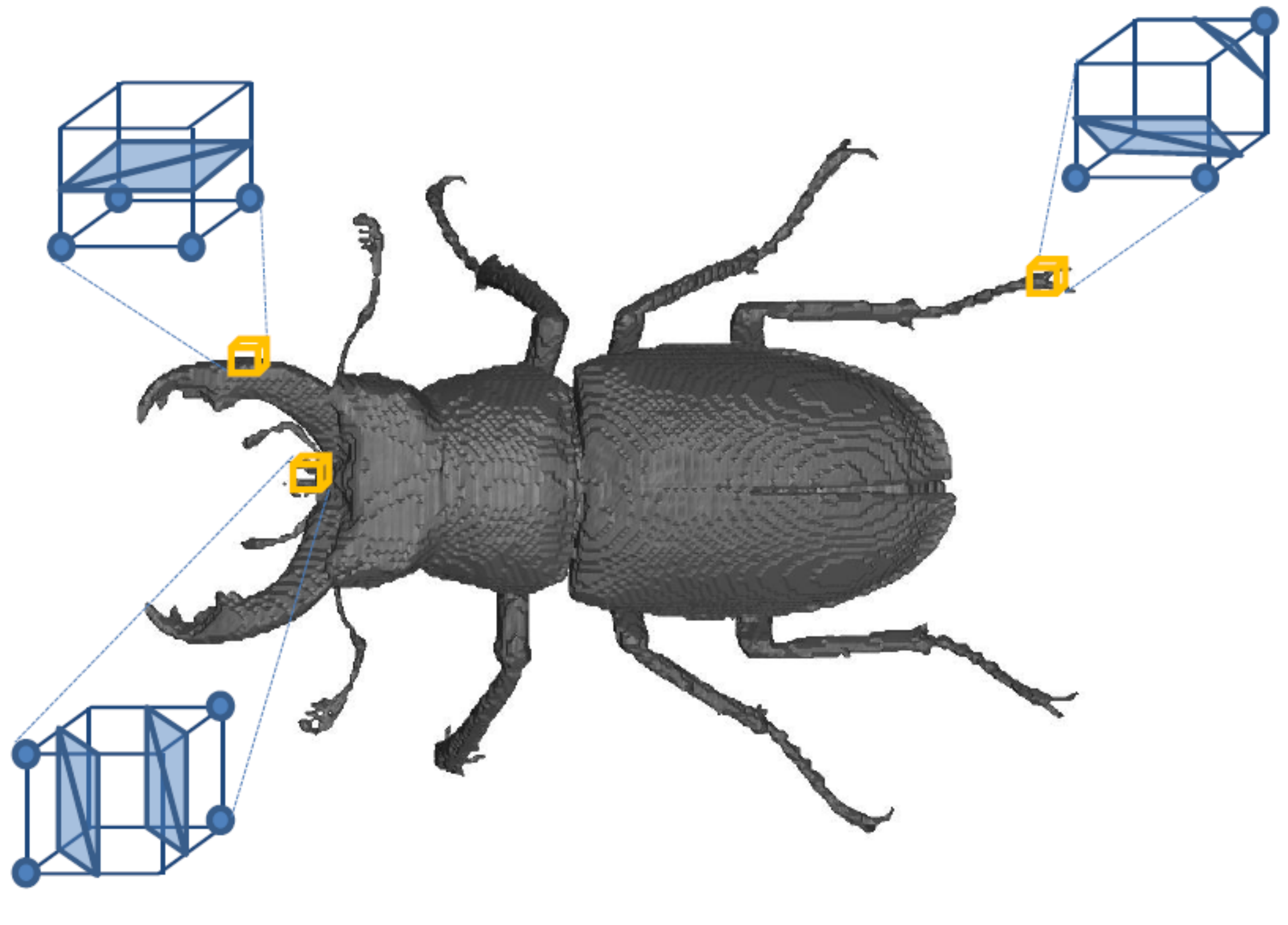


MSA: The Geometry Step (where on the edge?)



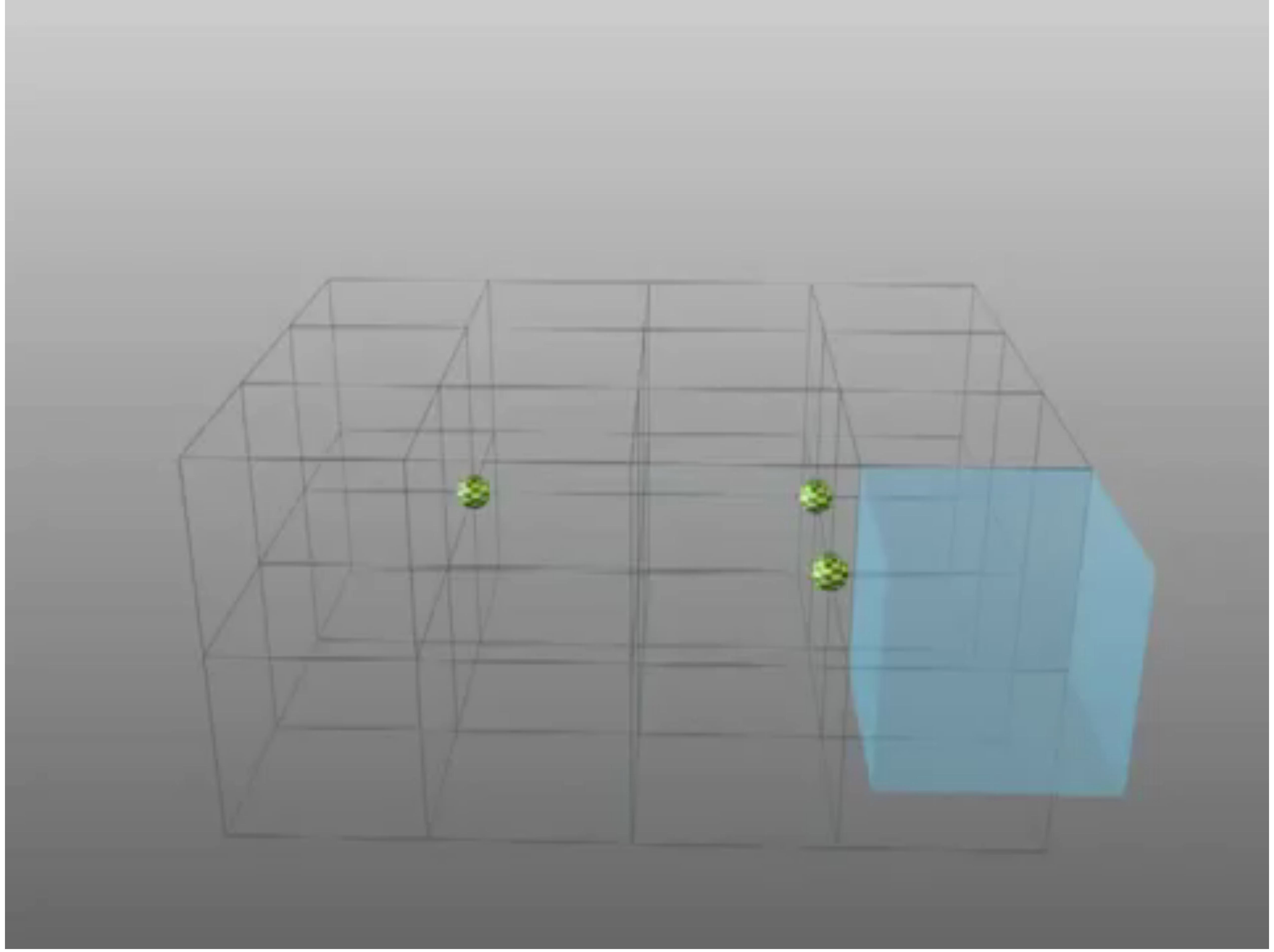
Marching Cubes Algorithm (MCA): Topological Cases





The Stag Beetle dataset is courtesy of Vienna University of Technology
https://www.cg.tuwien.ac.at/research/vis/datasets/

Marching Cubes Algorithm in Action!

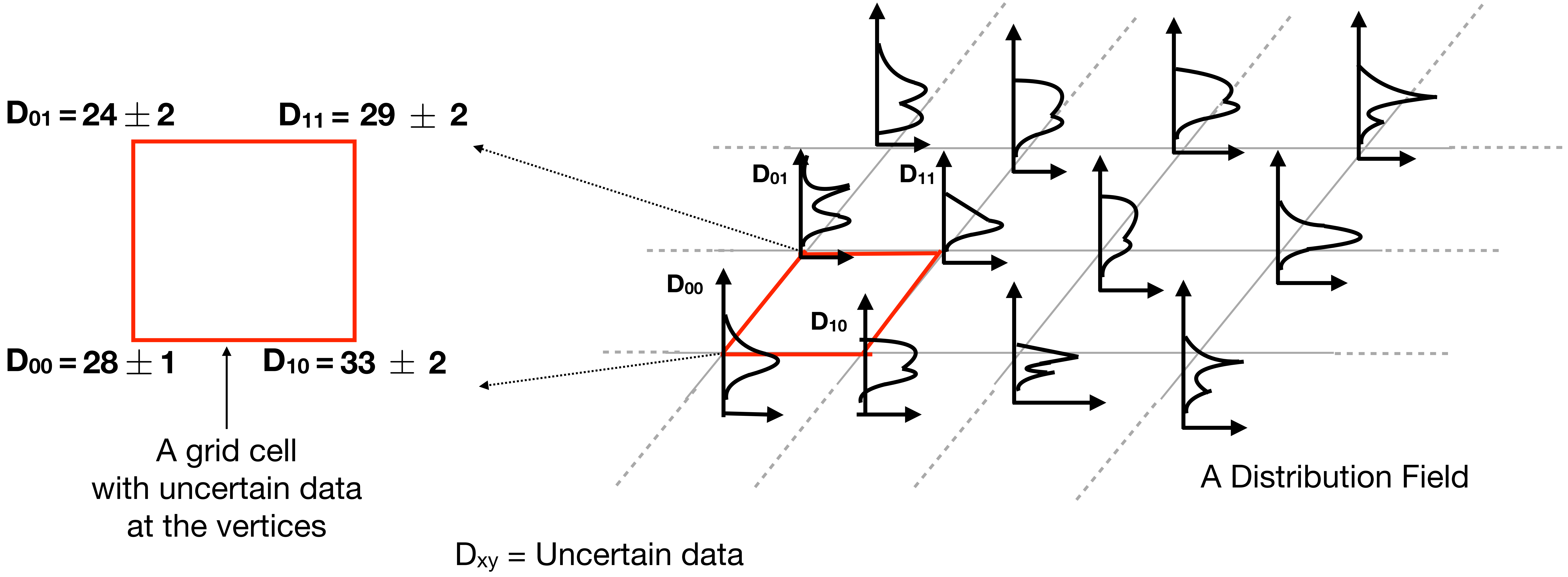


https://www.youtube.com/watch?v=LfttaAepYJ8

Marching Squares/Cubes Algorithm for Level-Set Extraction in Uncertain Data

MSA for Uncertain Data (our contribution!)

- Topological (which edges) uncertainty resolution
- Geometric (where on the edges) uncertainty resolution

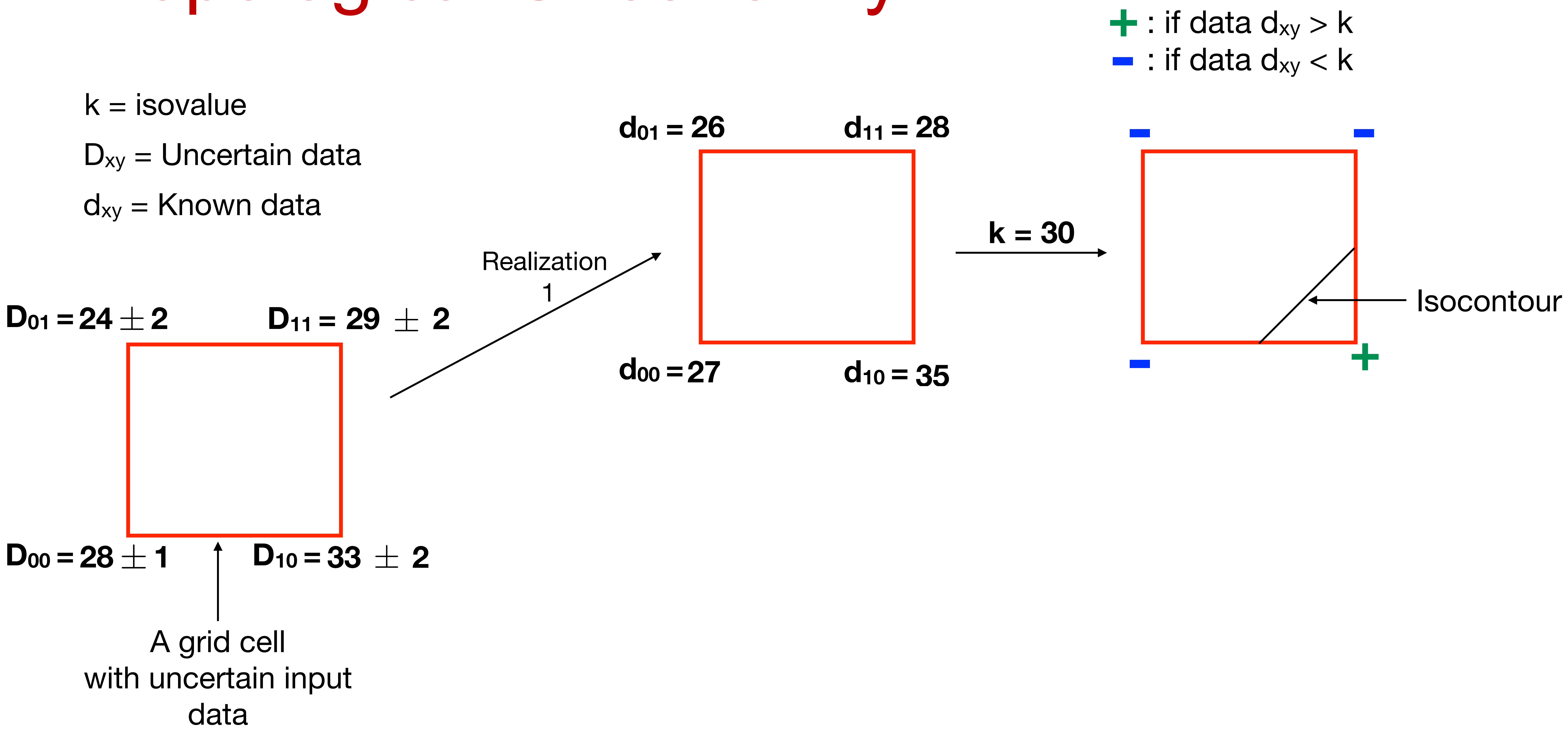


MSA: Topological Uncertainty

k = isovalue

D_{xy} = Uncertain data

d_{xy} = Known data



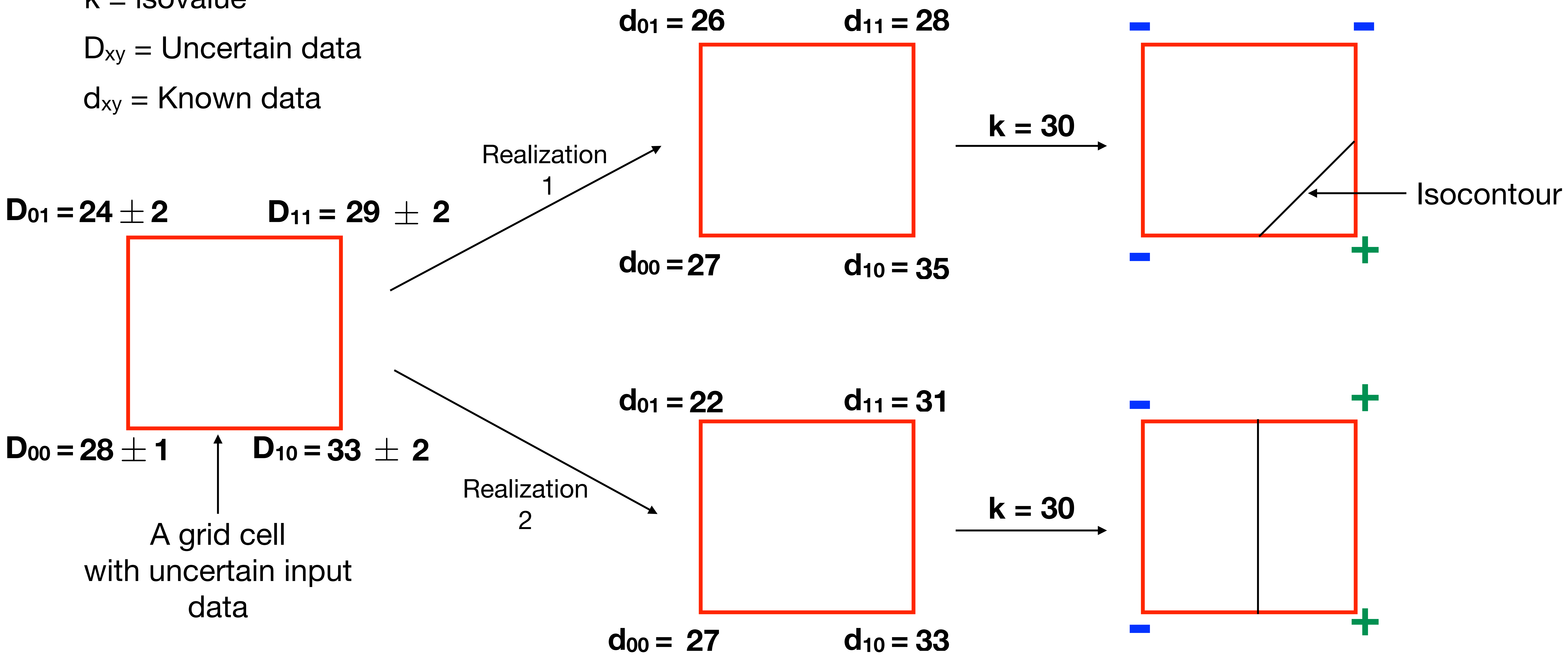
MSA: Topological Uncertainty

+ : if data $d_{xy} > k$
− : if data $d_{xy} < k$

k = isovalue

D_{xy} = Uncertain data

d_{xy} = Known data

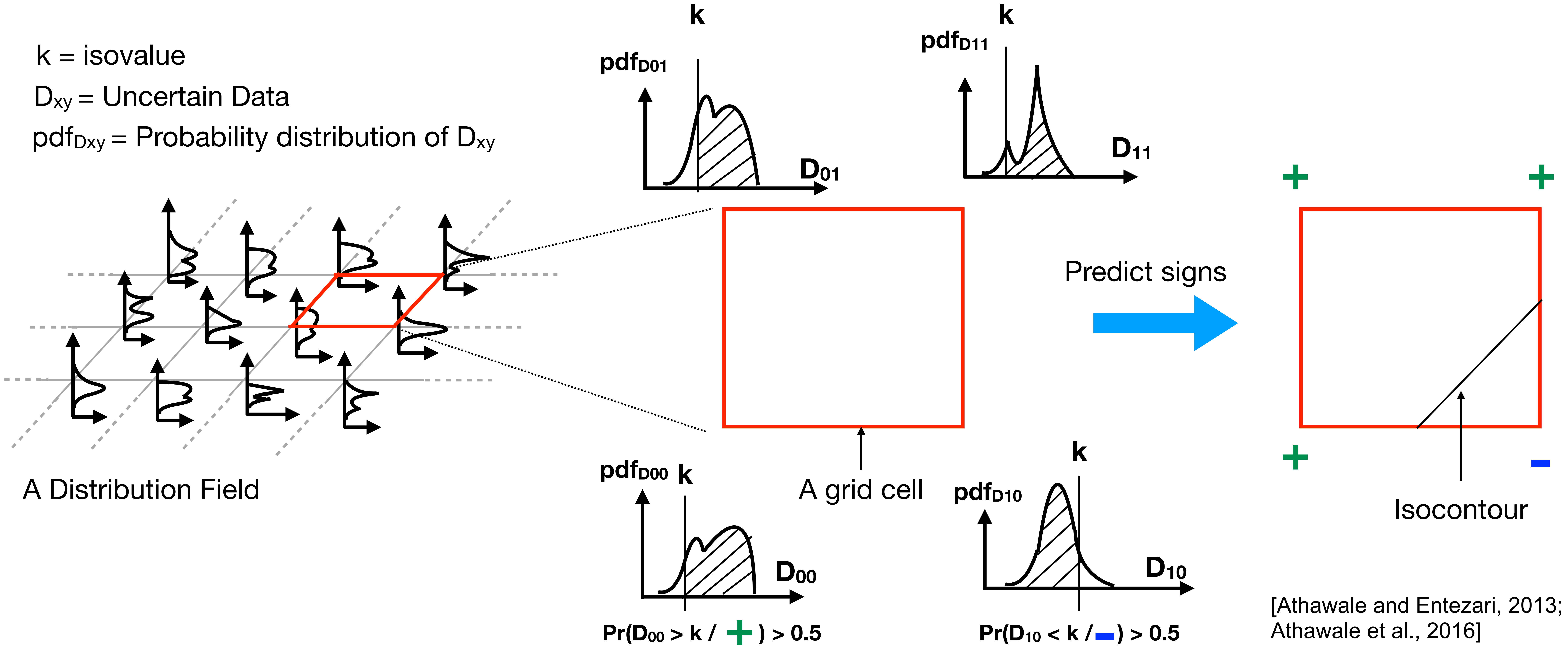


MSA: Topological Uncertainty Resolution

k = isovalue

D_{xy} = Uncertain Data

$pdf_{D_{xy}}$ = Probability distribution of D_{xy}



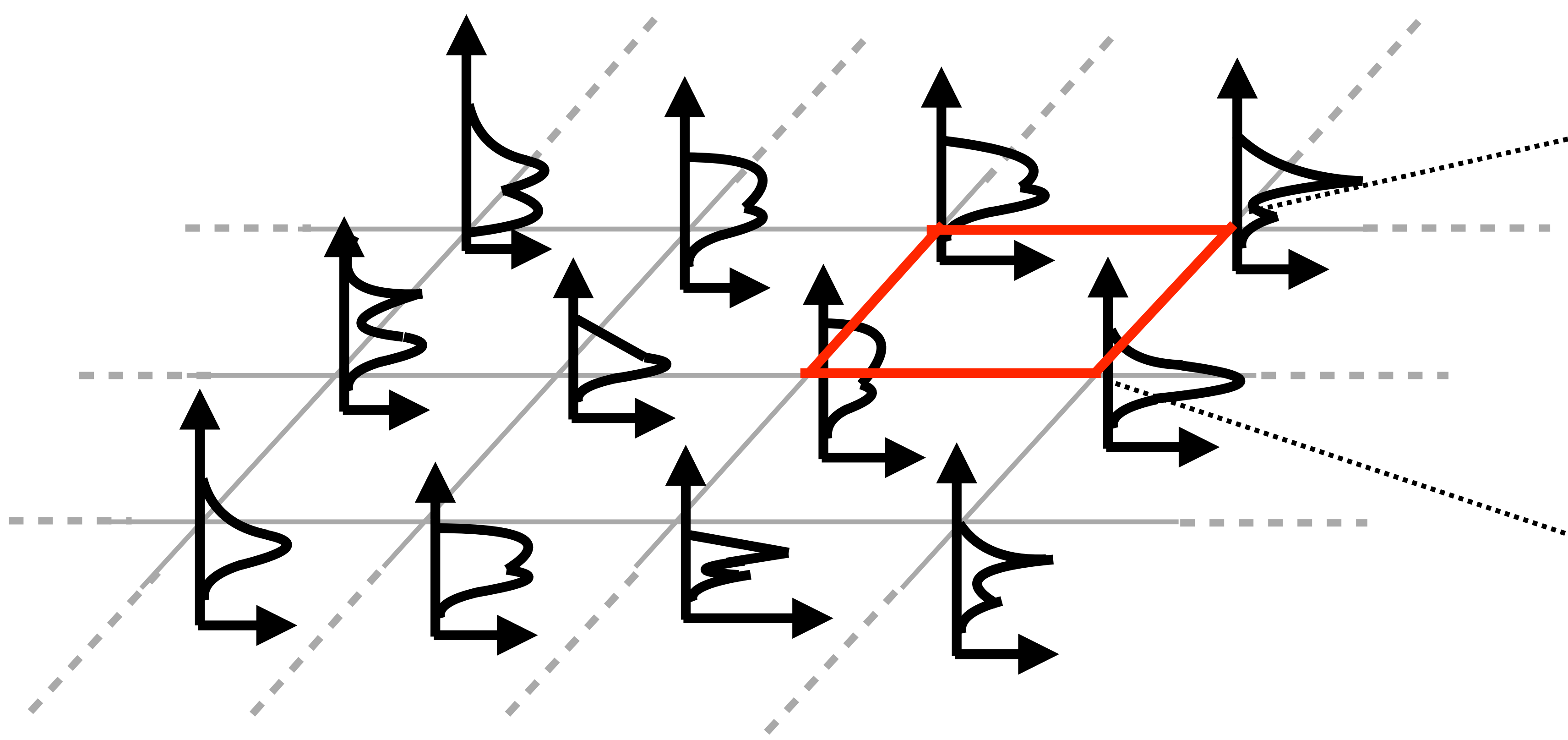
[Athawale and Entezari, 2013; Athawale et al., 2016]

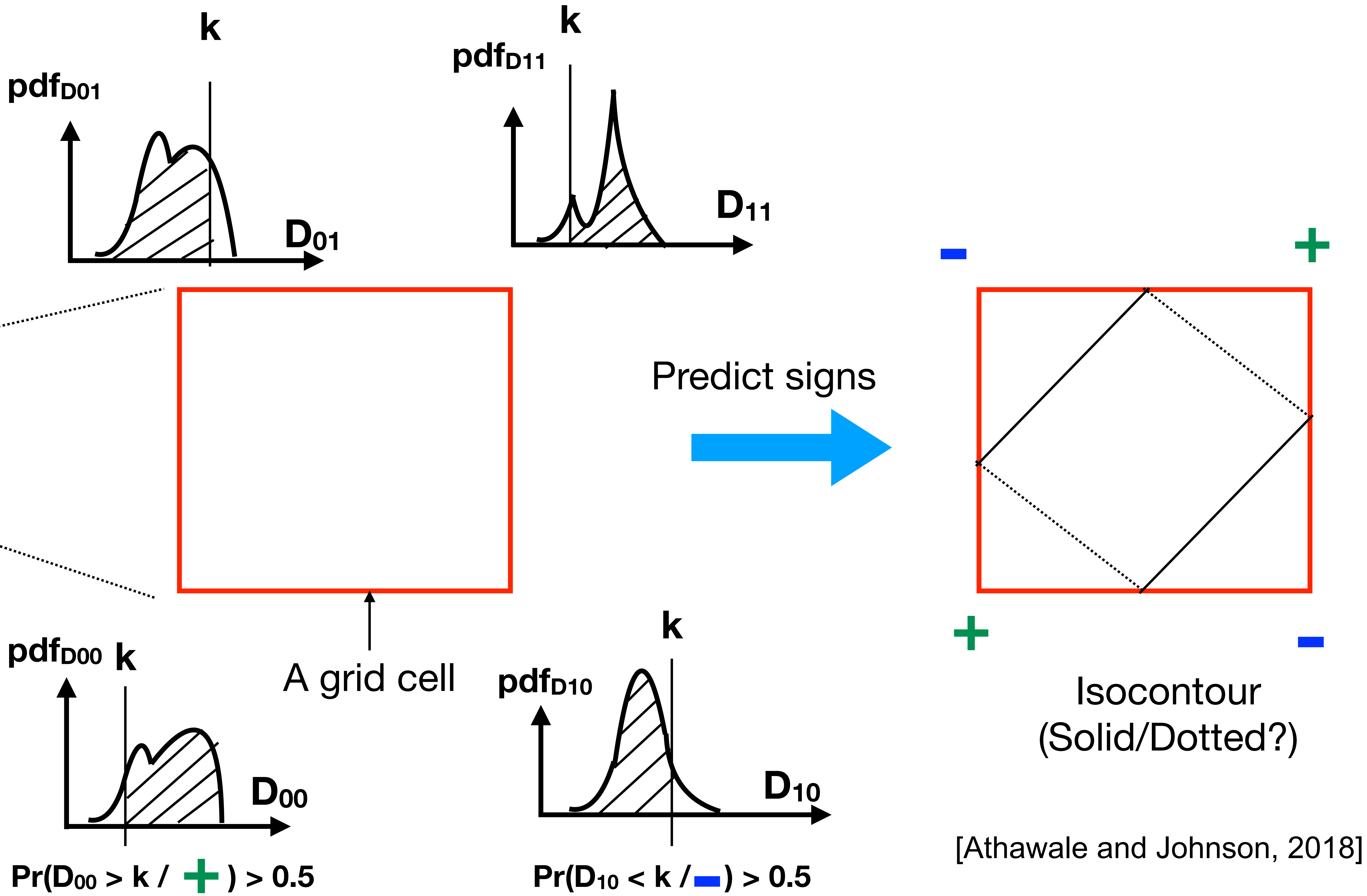
MSA Ambiguous Case: Topological Uncertainty Resolution

k = isovalue

D_{xy} = Uncertain Data

$pdf_{D_{xy}}$ = Probability distribution of D_{xy}





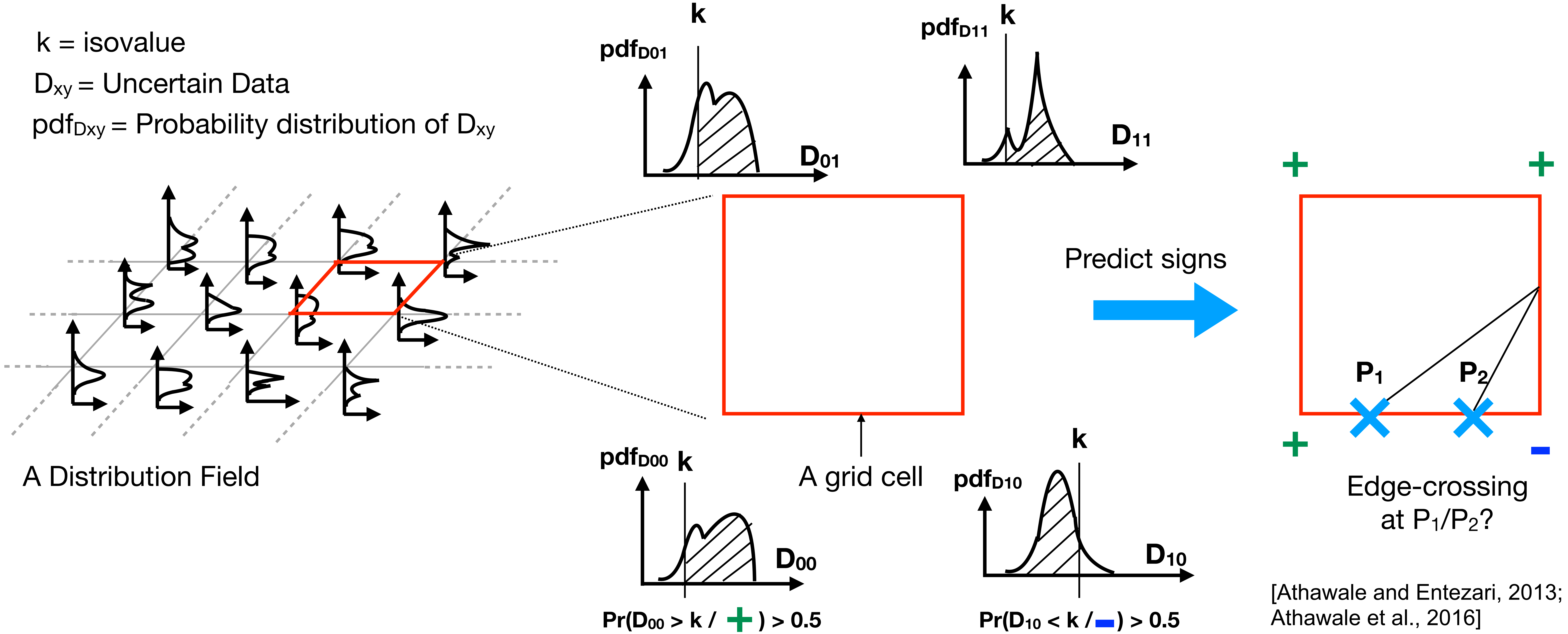
[Athawale and Johnson, 2018]

MSA: Geometric Uncertainty

k = isovalue

D_{xy} = Uncertain Data

$pdf_{D_{xy}}$ = Probability distribution of D_{xy}

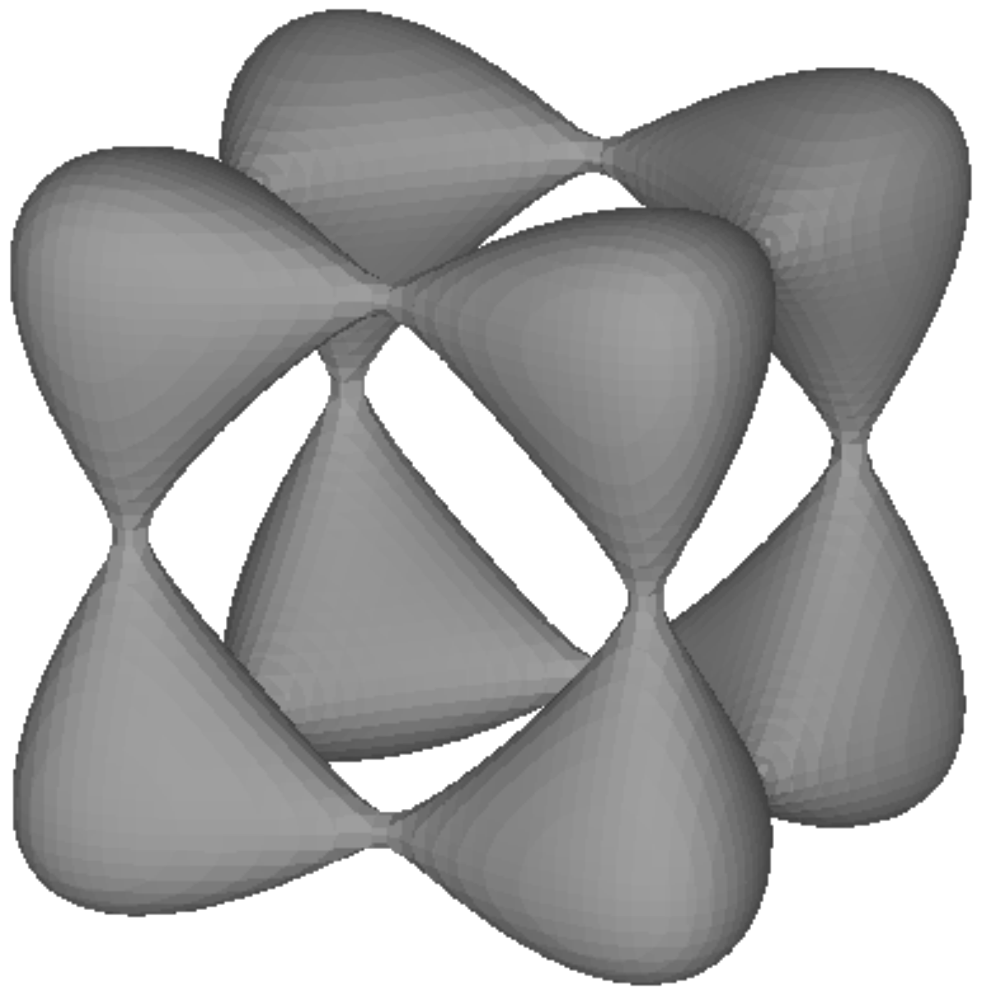


[Athawale and Entezari, 2013; Athawale et al., 2016]

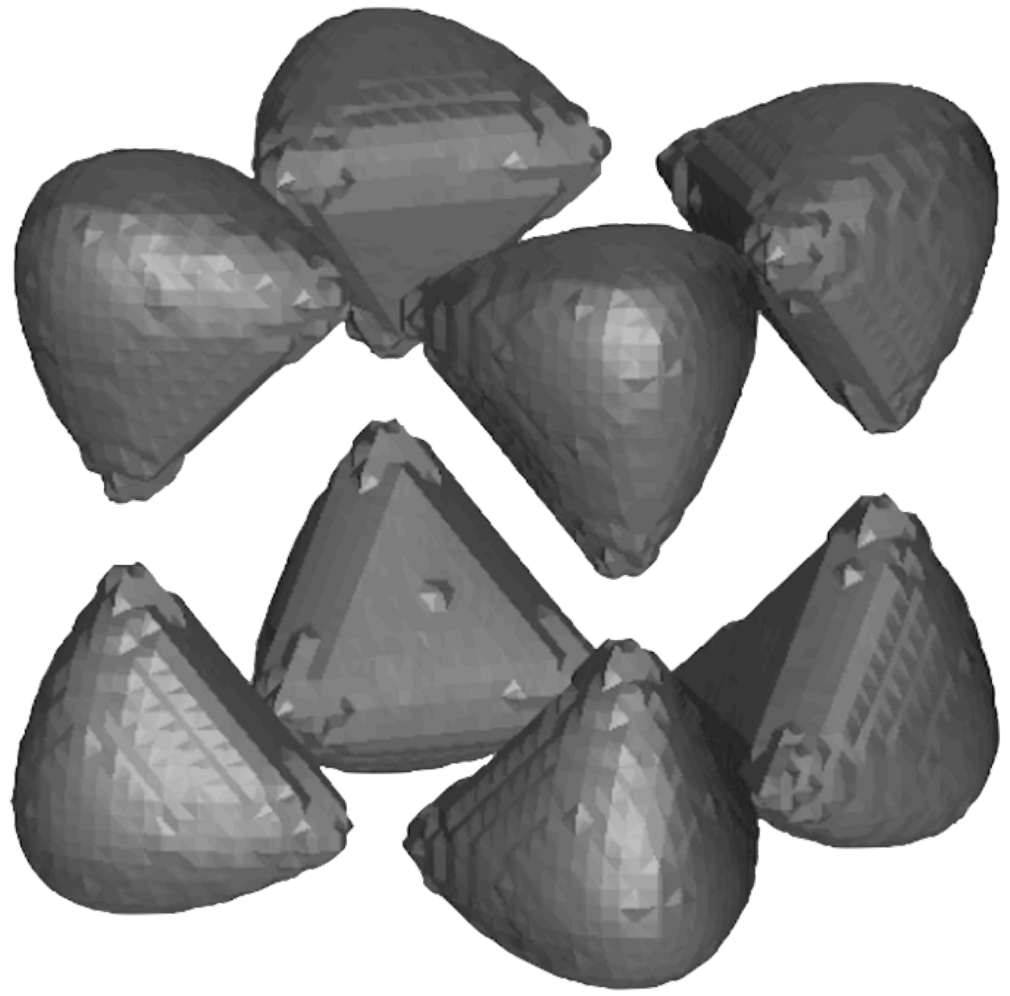
Results: Visualization of Uncertain Level Sets

Isosurface Extraction in Uncertain Data

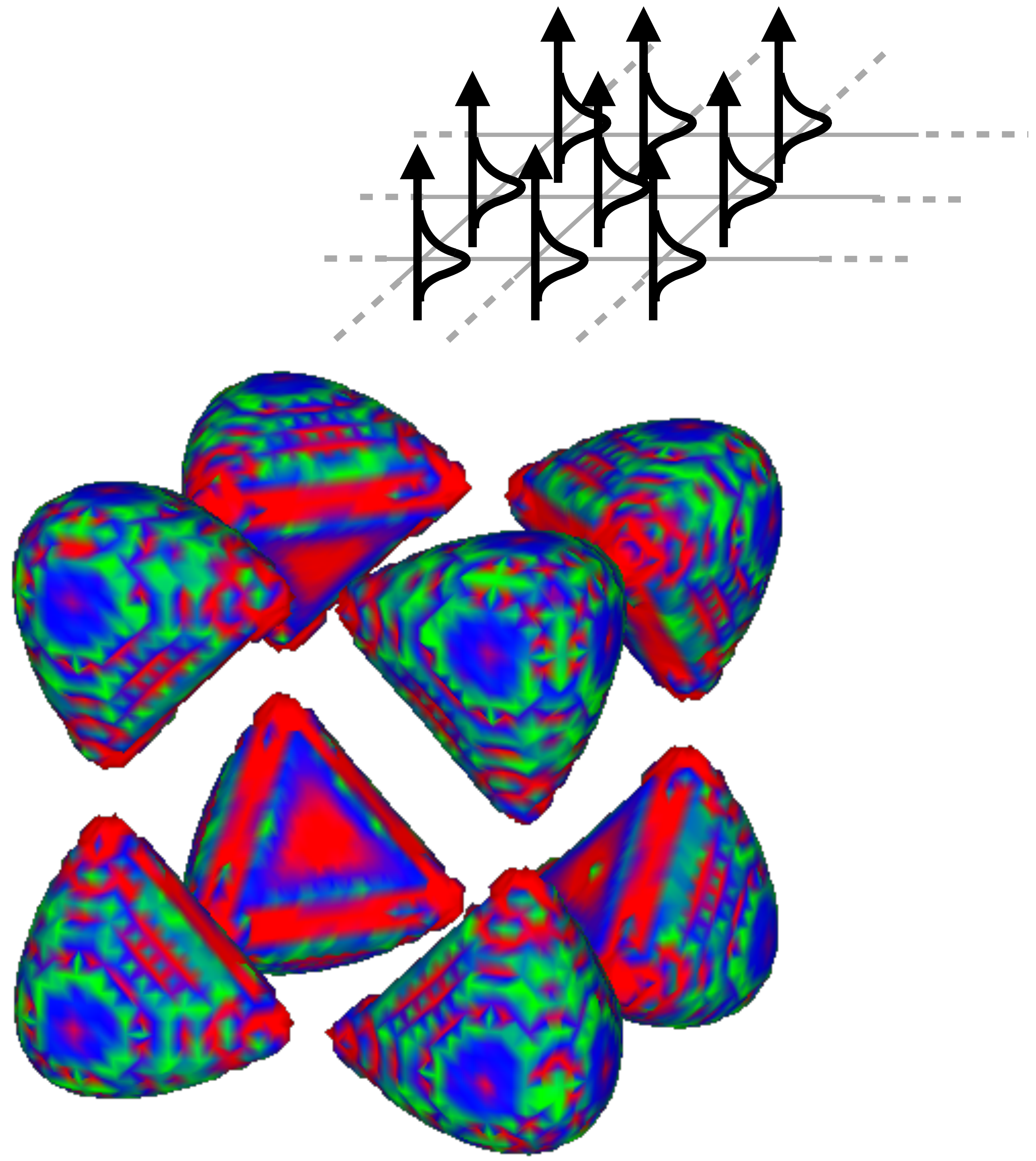
Tangle function (synthetic data)



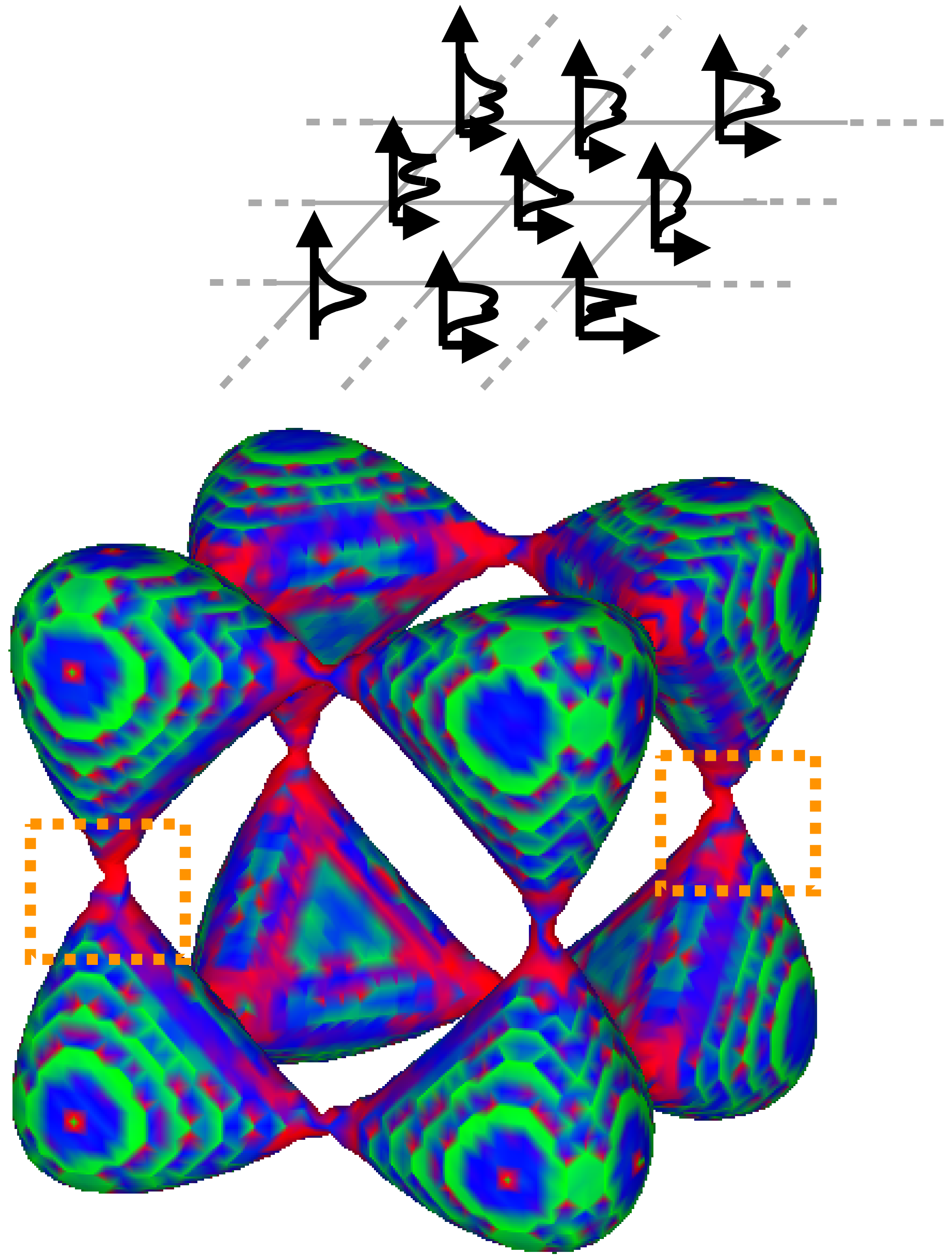
Ground truth



Mean Field



Parametric Distribution Field



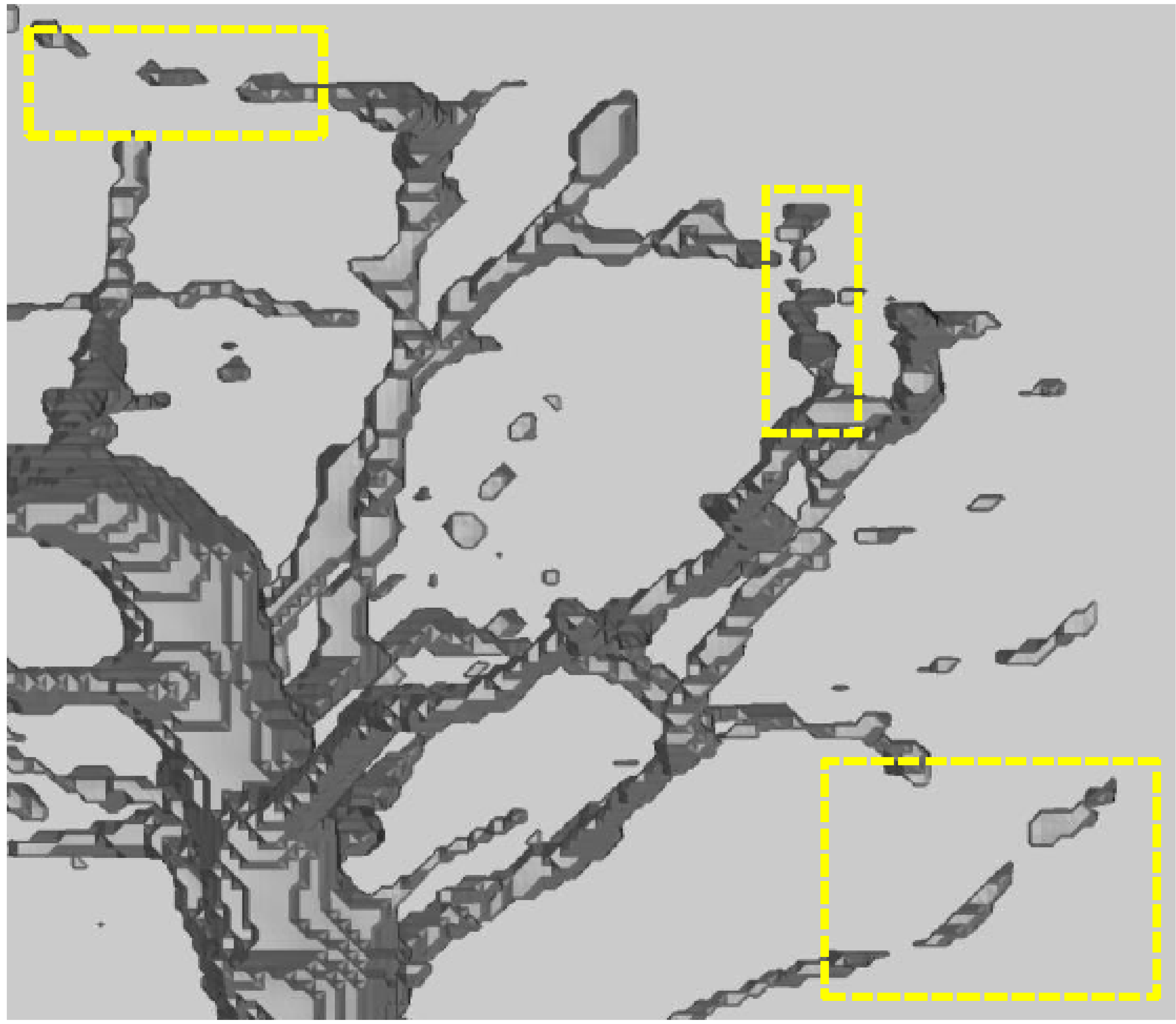
Nonparametric Distribution Field

[Athawale and Entezari, 2013; Athawale et al., 2016]

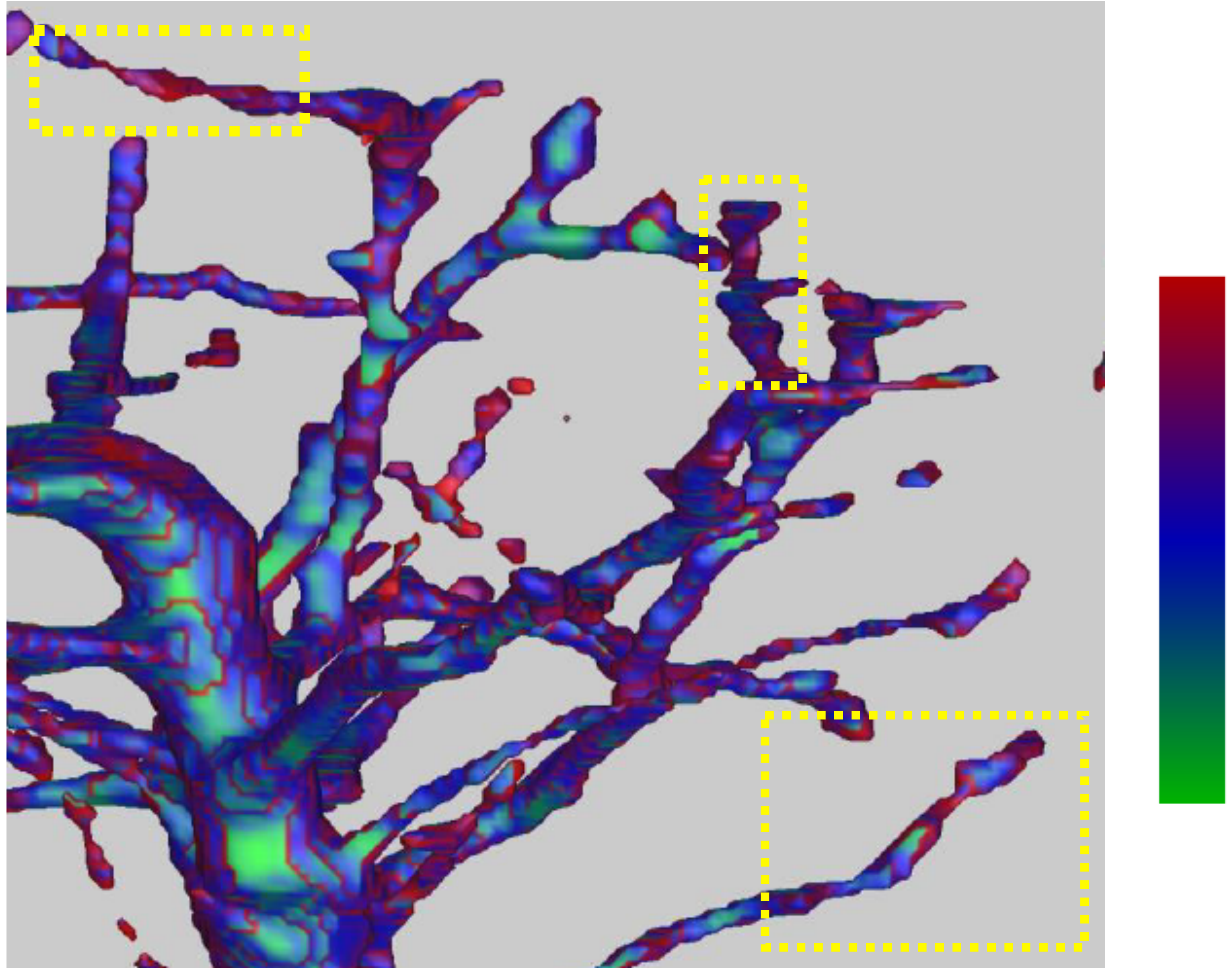
(Visualization software: The Geomview, Developer: The Geometry Center at the University of Minnesota)

Isosurface Extraction in Uncertain Data

Bonsai tree (real data)



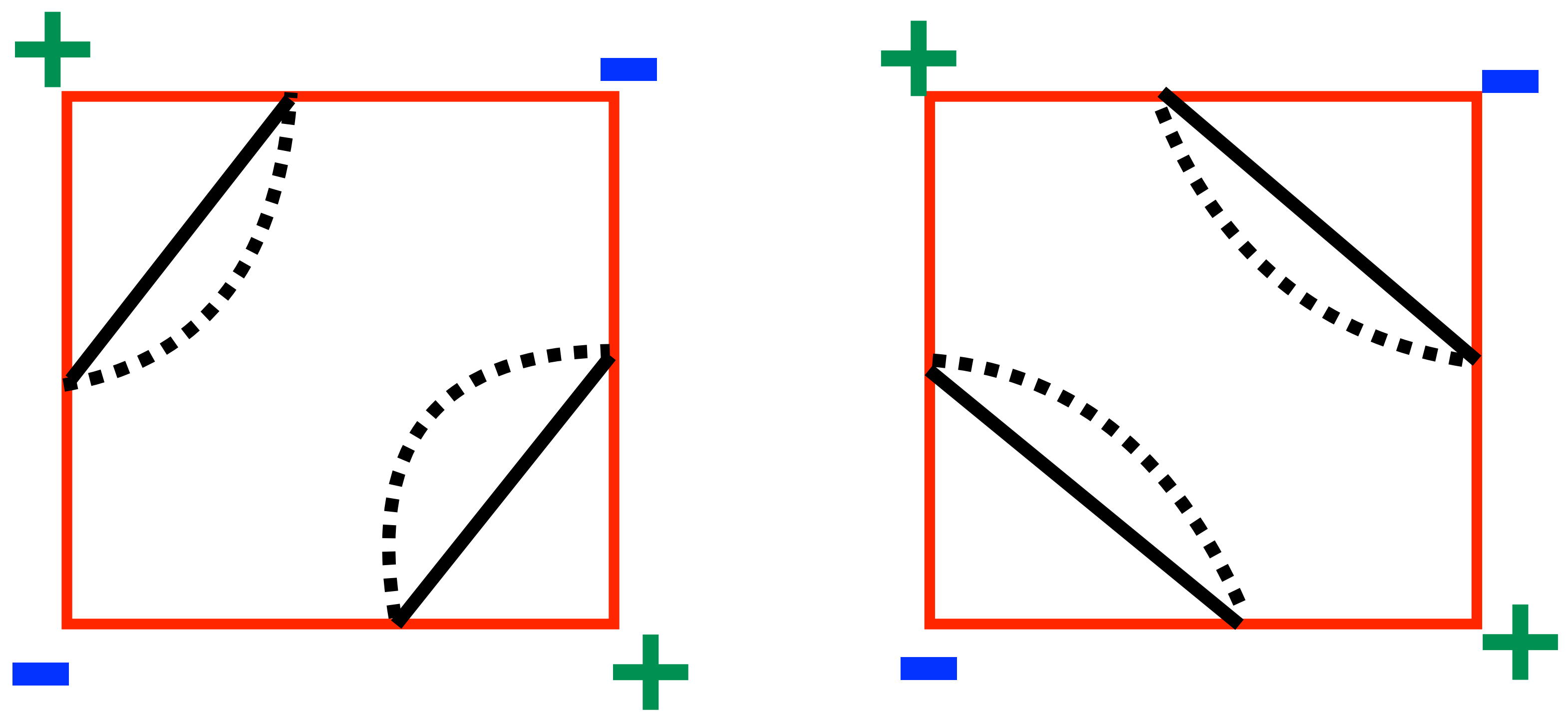
Mean



Nonparametric

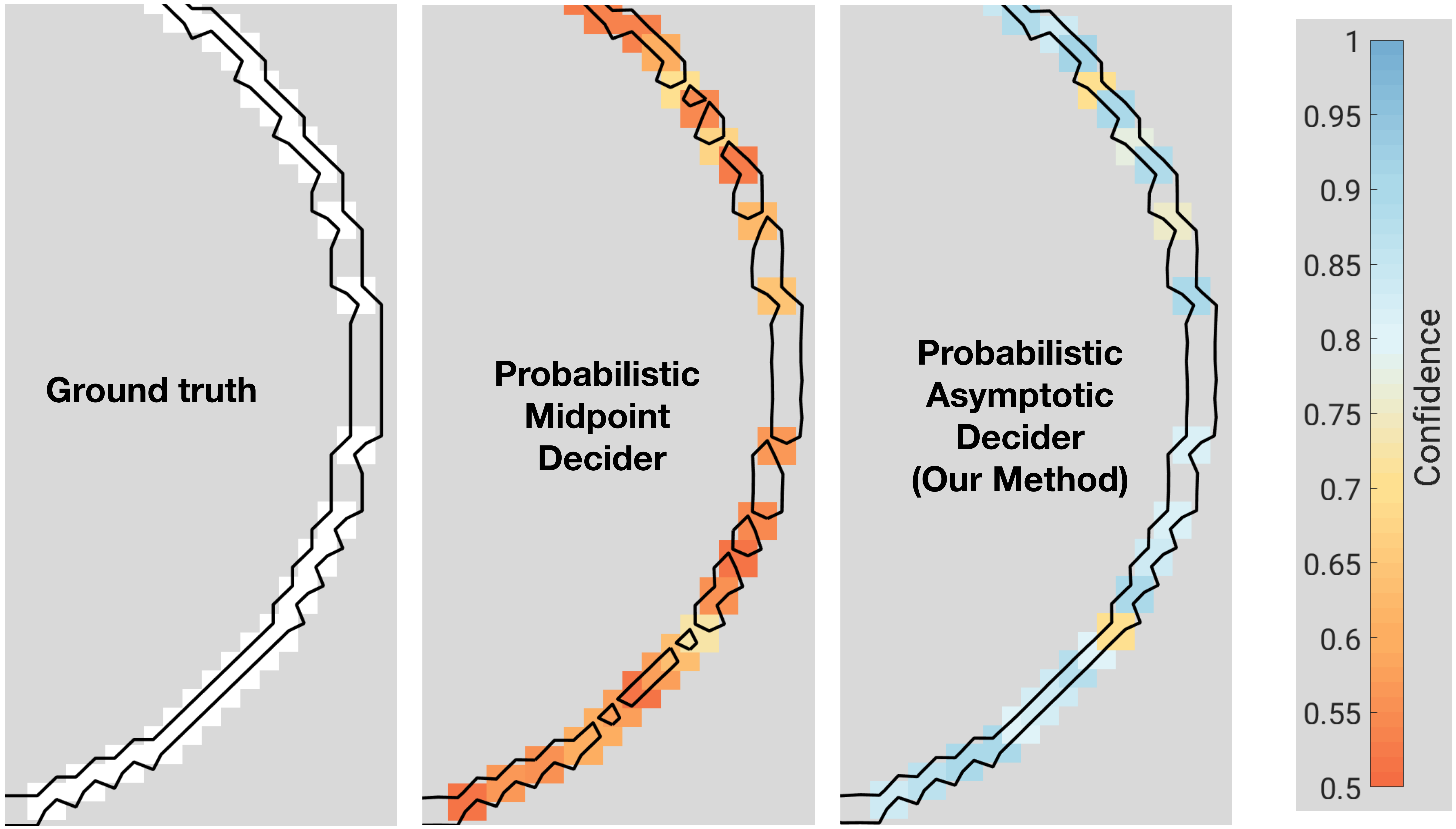
MSA Ambiguous Case Resolution in Uncertain Data

Concentric circles (synthetic data)

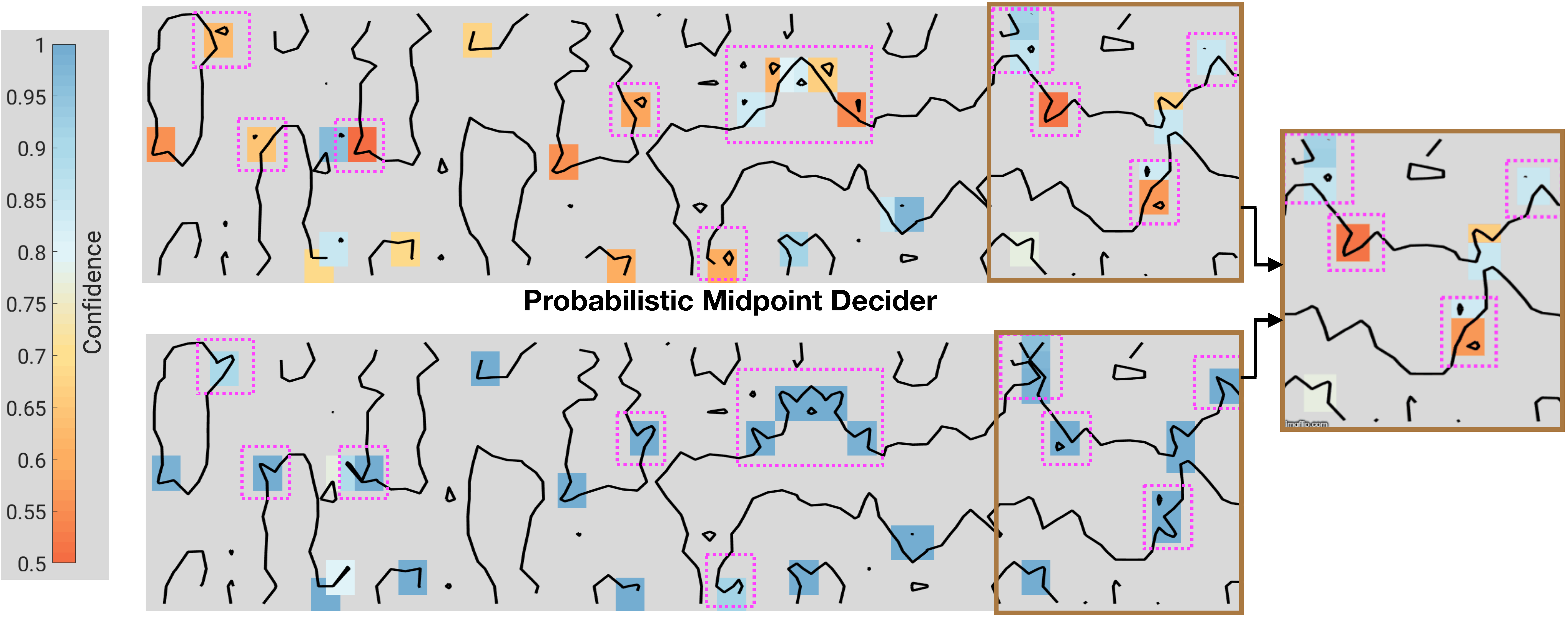


MSA ambiguous case

[Athawale and Johnson, 2018]



Isocontour Visualizations (Kàrmàn Vortex Street)



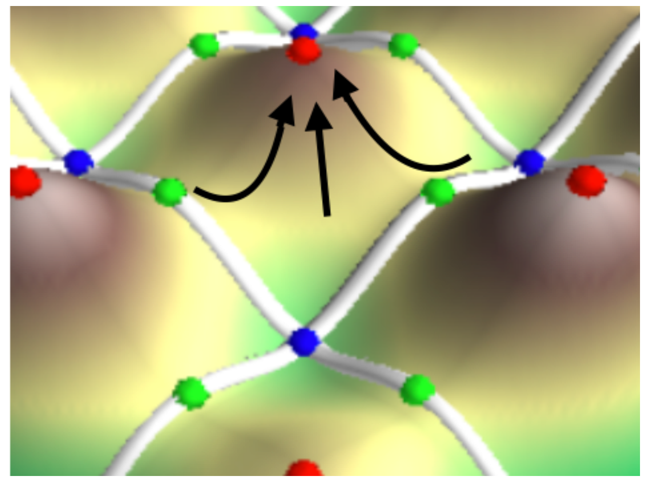
Probabilistic Midpoint Decider

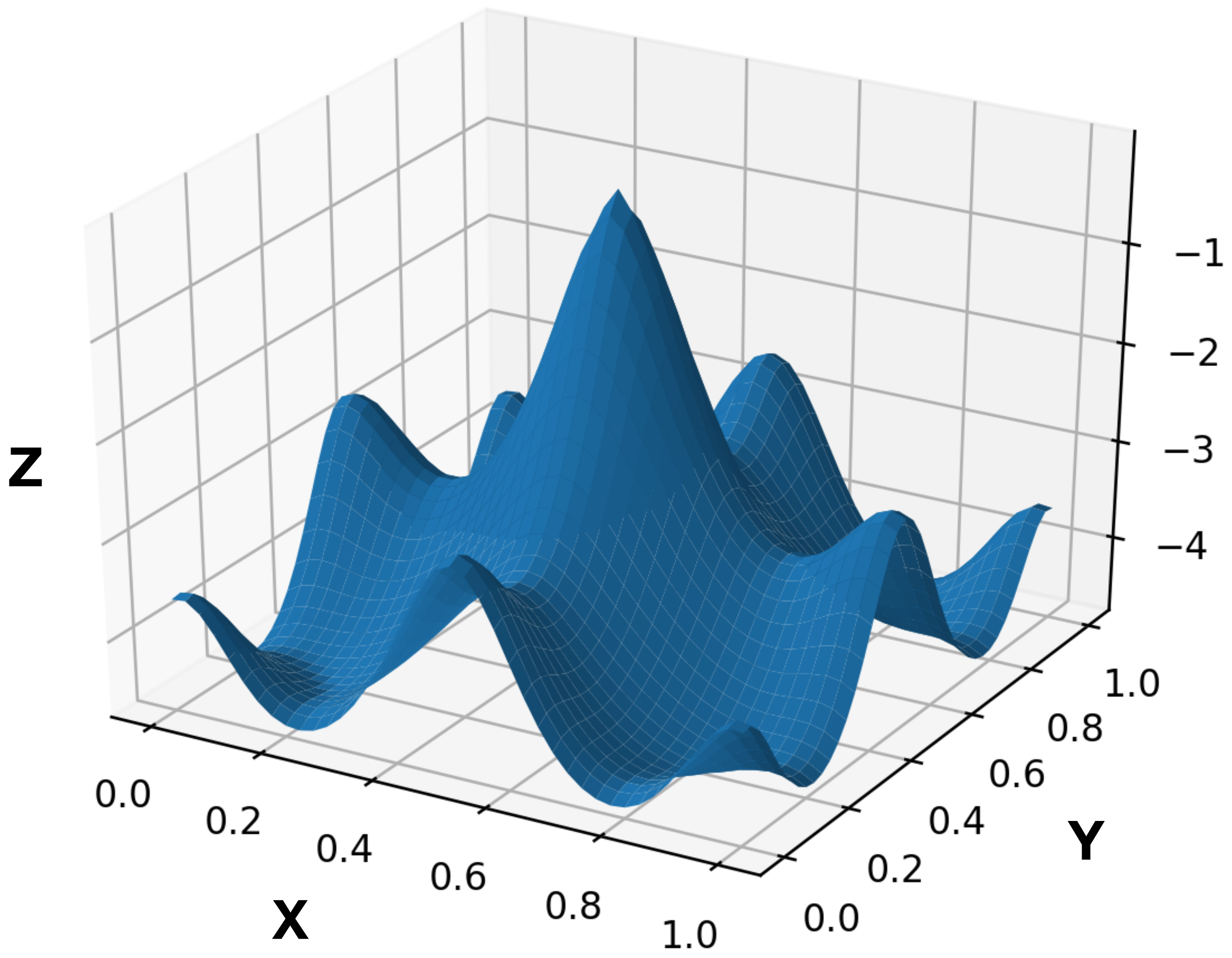
Probabilistic Asymptotic Decider (Our Method)

The flow simulation dataset is courtesy of the Gerris project [Popinet, 2003]

Morse Complex Visualizations

Topological descriptors, which provide an abstract representation of gradient flows of a scalar field



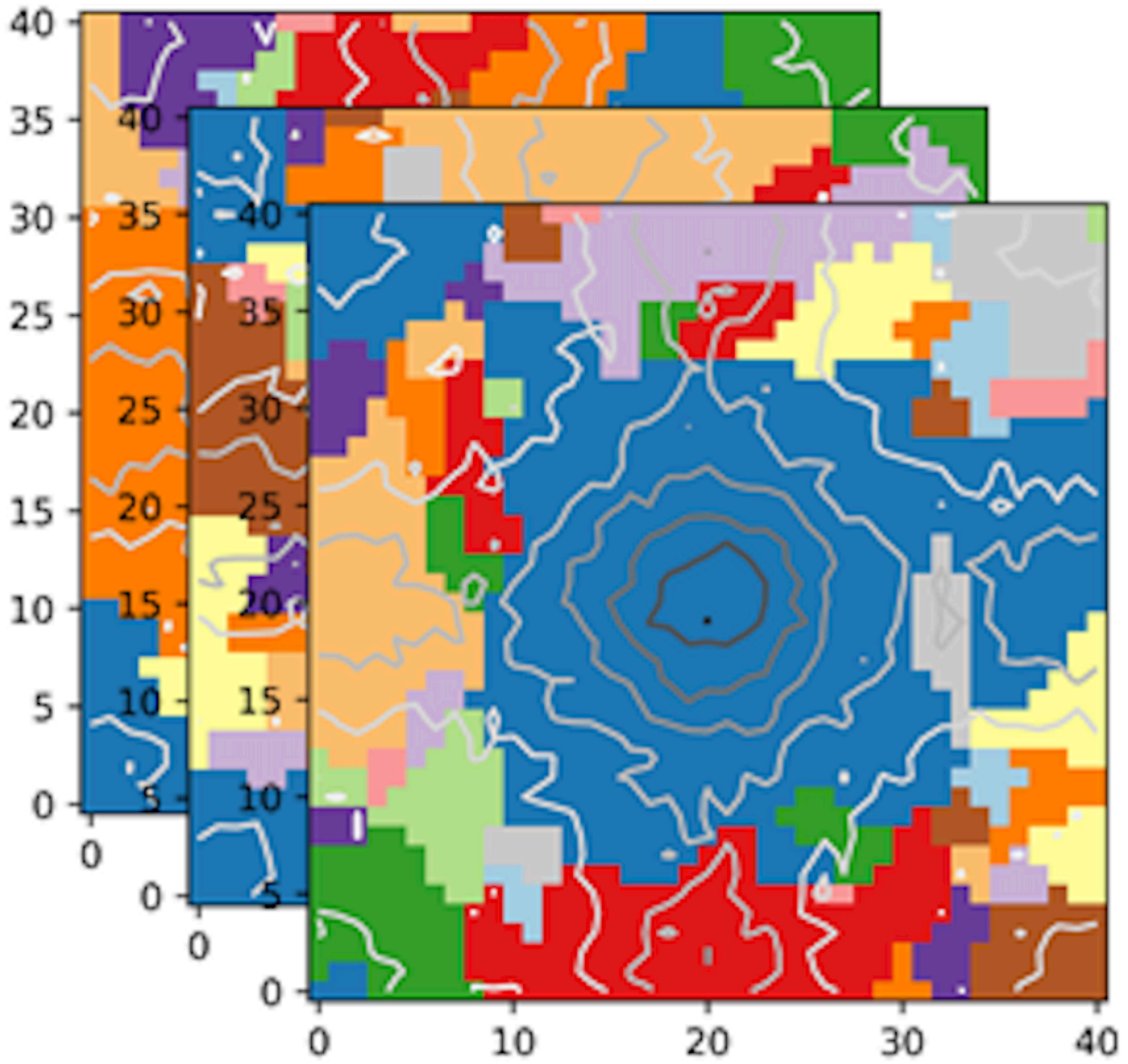


The Ackley function
[Ackley D. H., 1987]

Morse complex visualization
of the Ackley function

Uncertainty Visualization of Morse Complexes

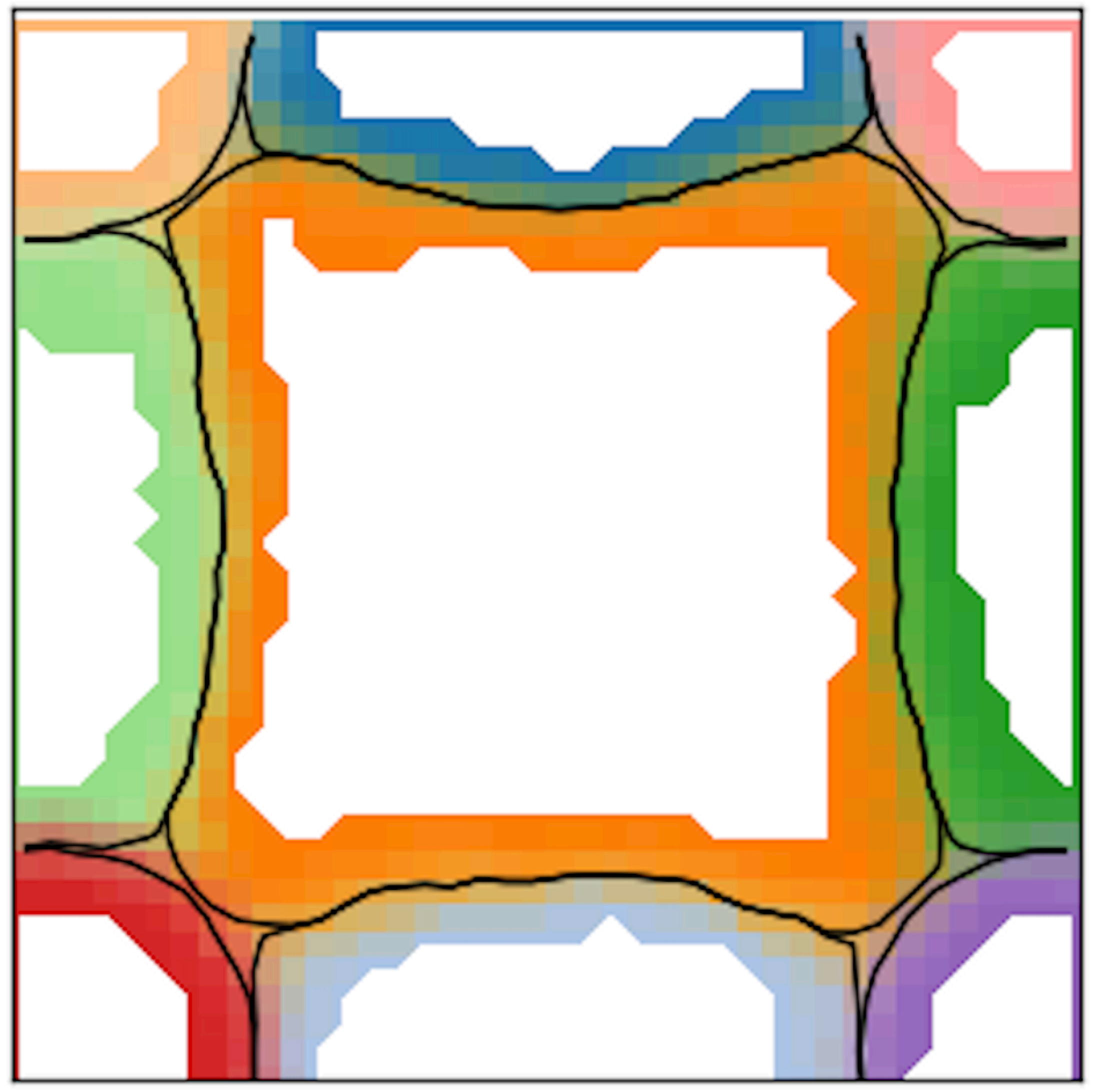
Visualize agreement/certainty and disagreement/uncertainty among abstract Morse complexes for ensembles



Ensemble of Morse Complexes

Agreement Regions

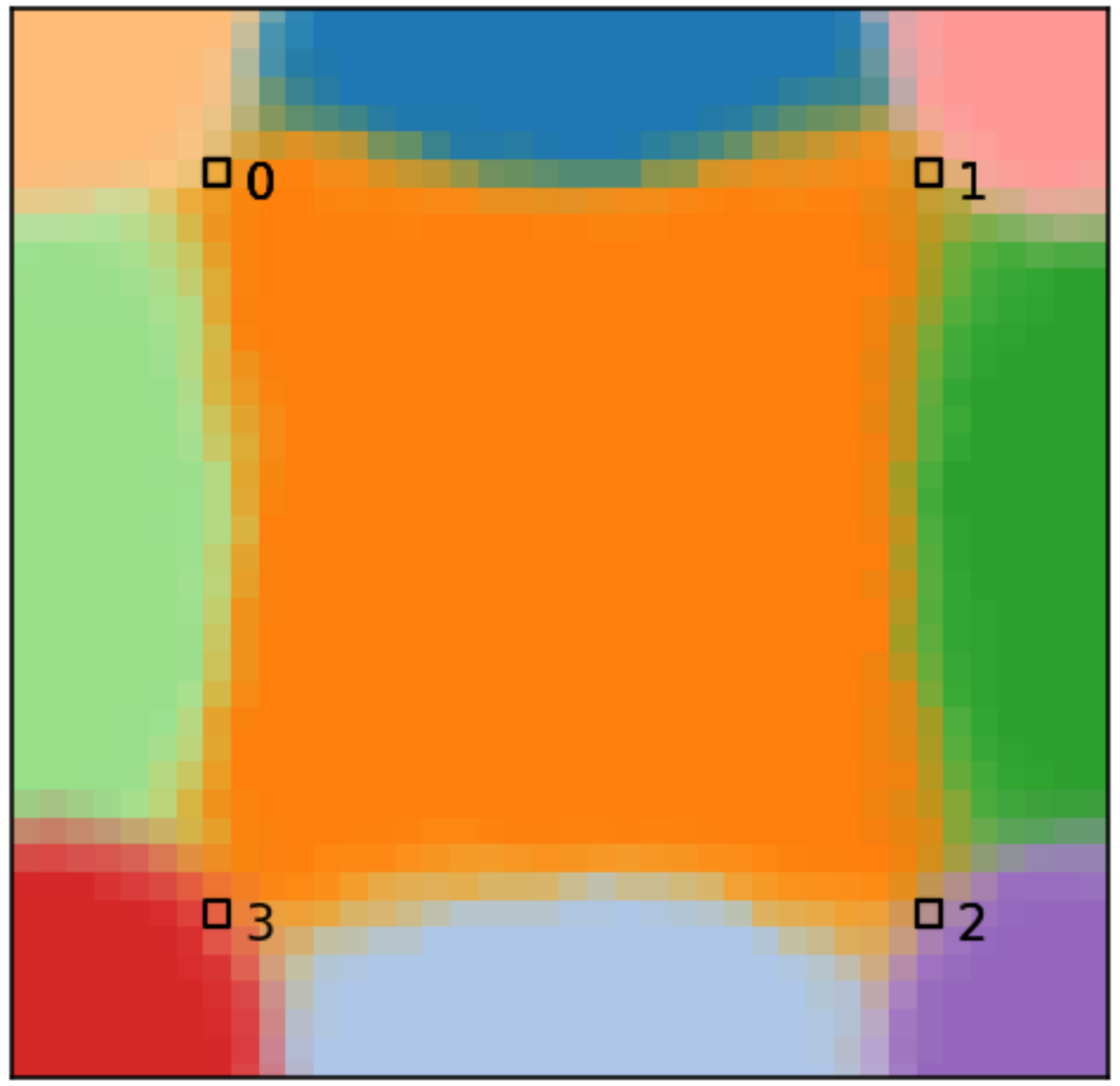
+

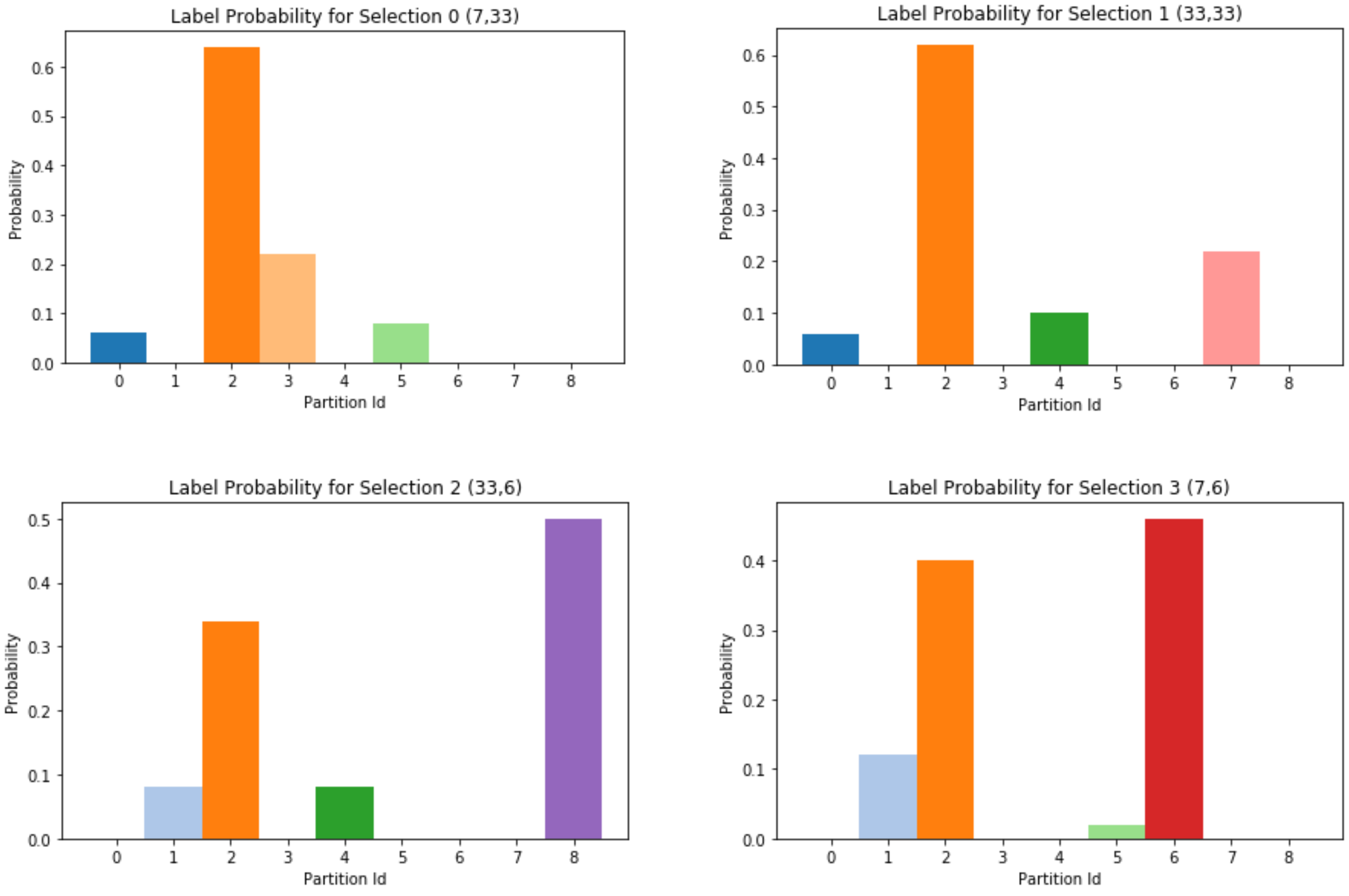


Uncertainty Regions

T. M. Athawale, D. Maljovec, C. R. Johnson, V. Pascucci, and B. Wang; **Uncertainty Visualization of 2D Morse Complex Ensembles using Statistical Summary Maps** (in progress).

Interactive PDF Queries for Uncertain Regions

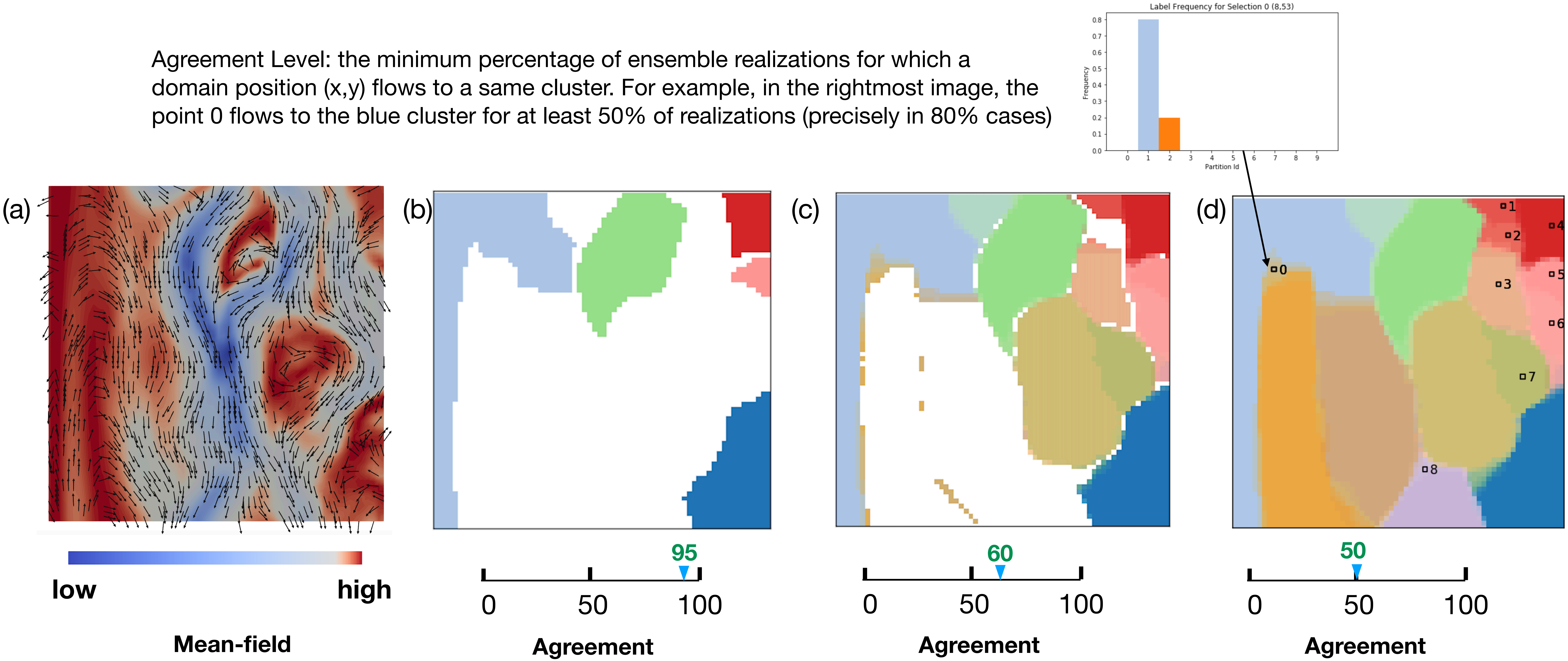




[K. Potter, R. M. Kirby, D. Xiu, and C. R. Johnson; Interactive Visualization of Probability and Cumulative Density Functions; 2011]

Uncertainty-Aware Morse Complex Visualizations

Agreement Level: the minimum percentage of ensemble realizations for which a domain position (x,y) flows to a same cluster. For example, in the rightmost image, the point 0 flows to the blue cluster for at least 50% of realizations (precisely in 80% cases)



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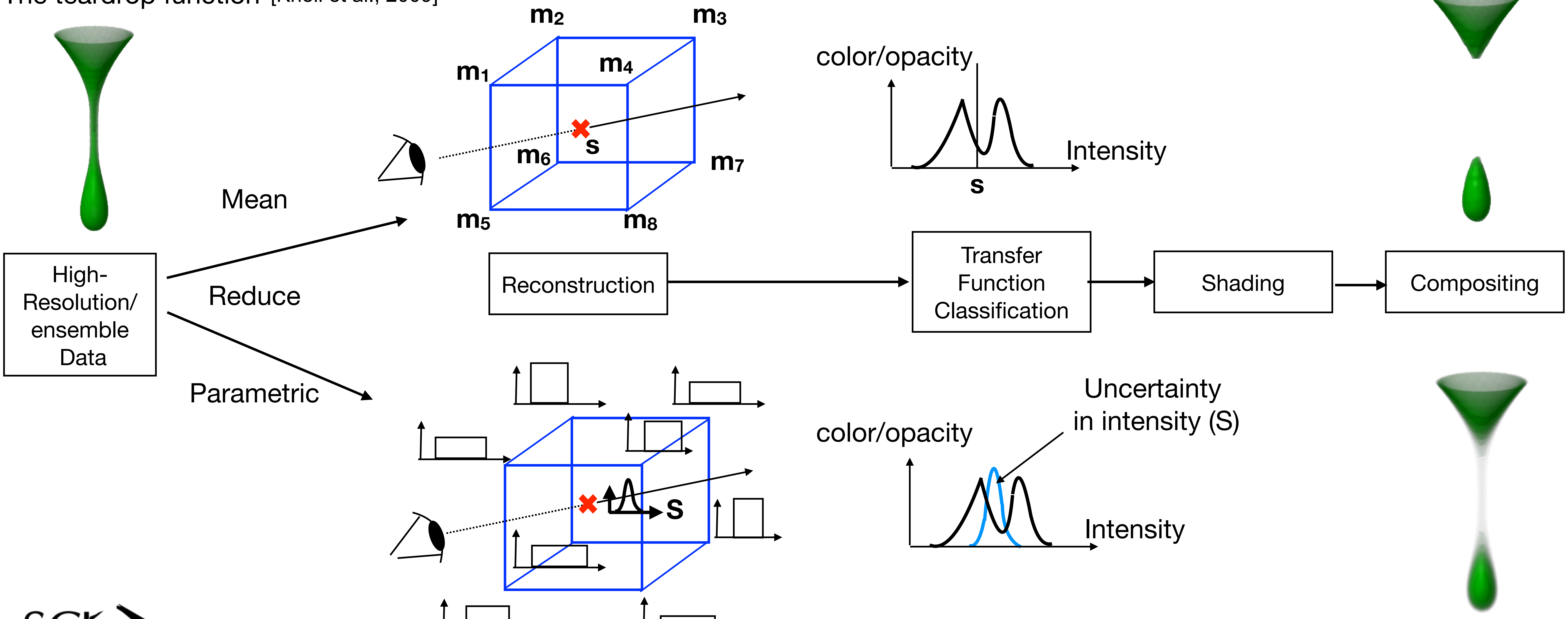
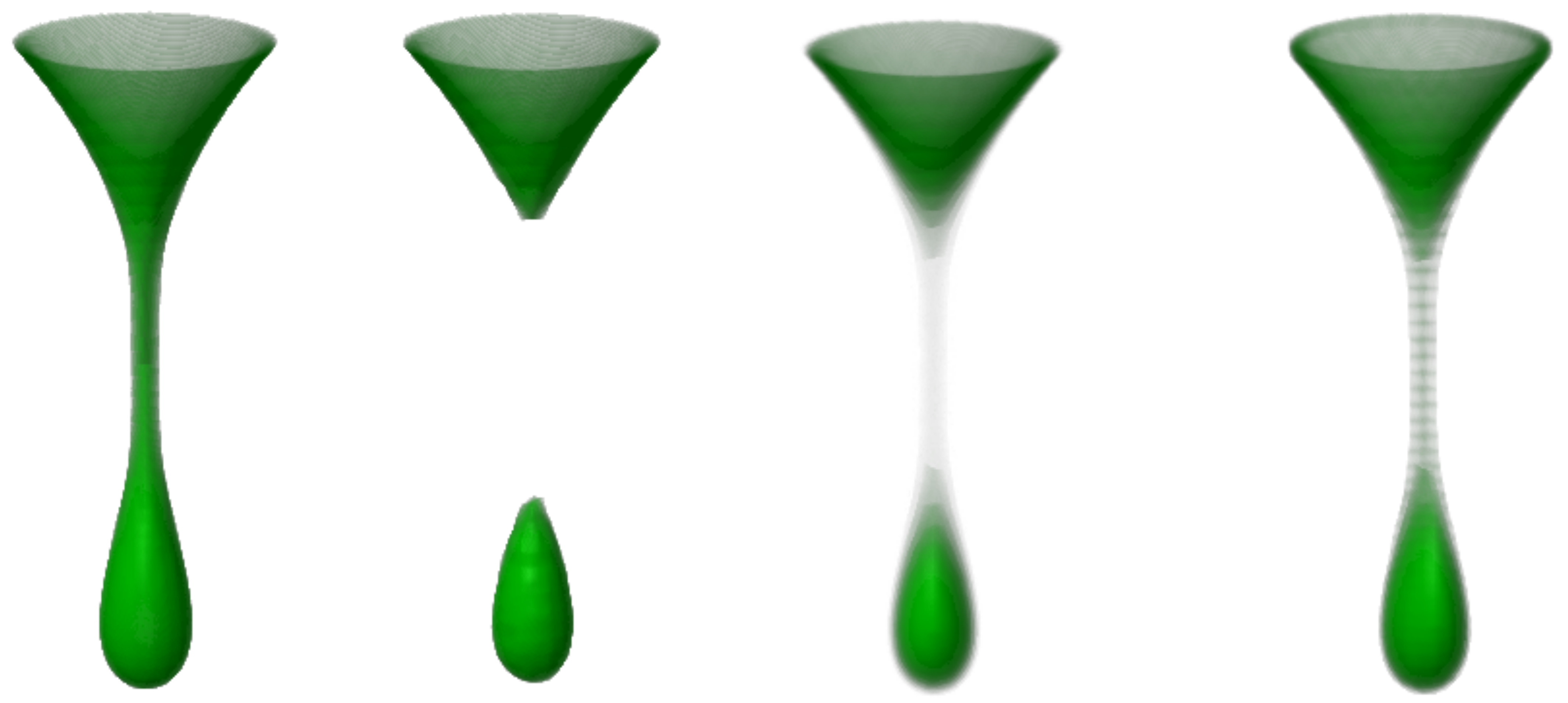
Direct Volume Rendering of Uncertain Data

[Liu et al., 2012; Sakhaee and Entezari, 2017]

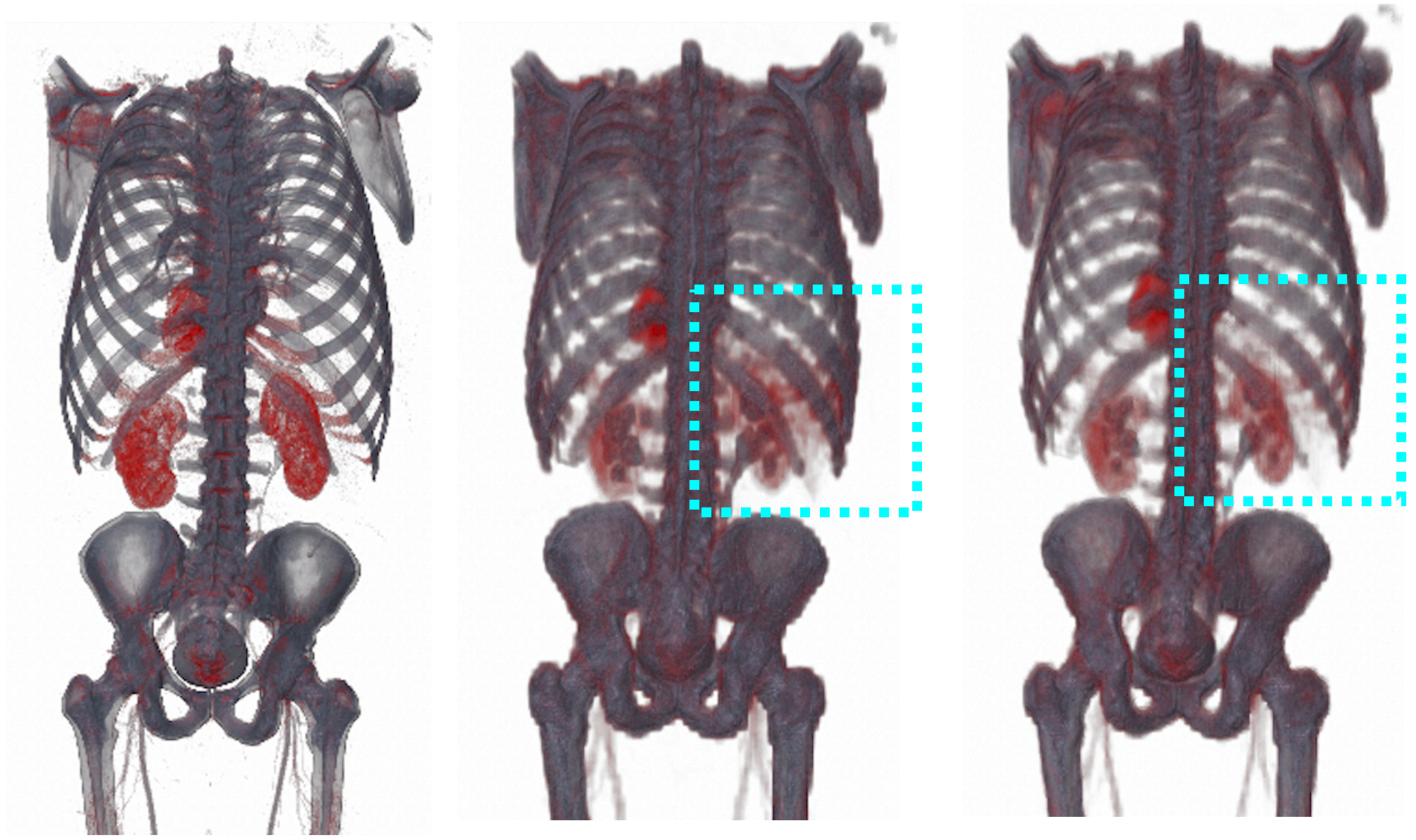
The teardrop function [Knoll et al., 2009]



DVR of Uncertain Data (Nonparametric)



The teardrop function

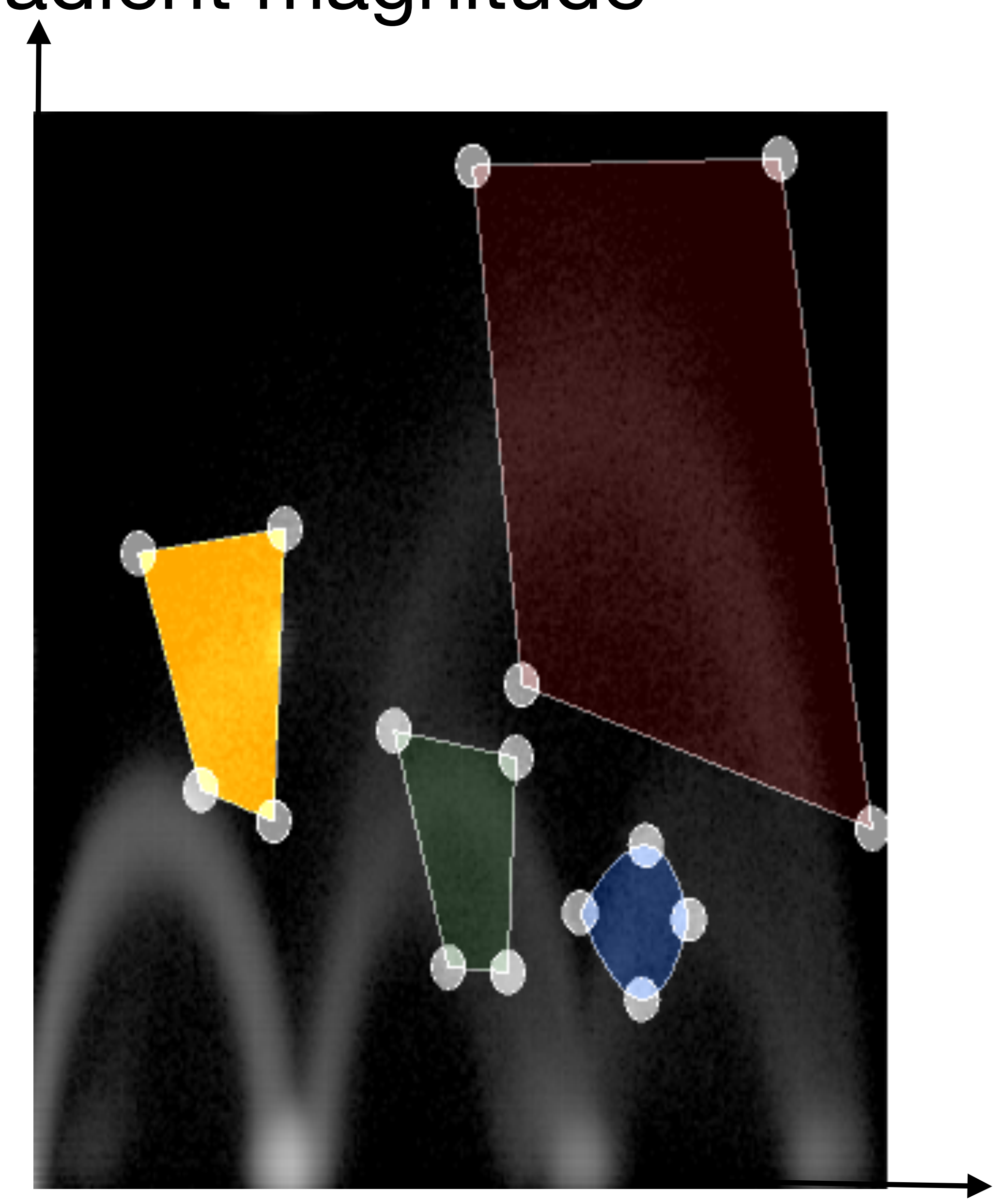


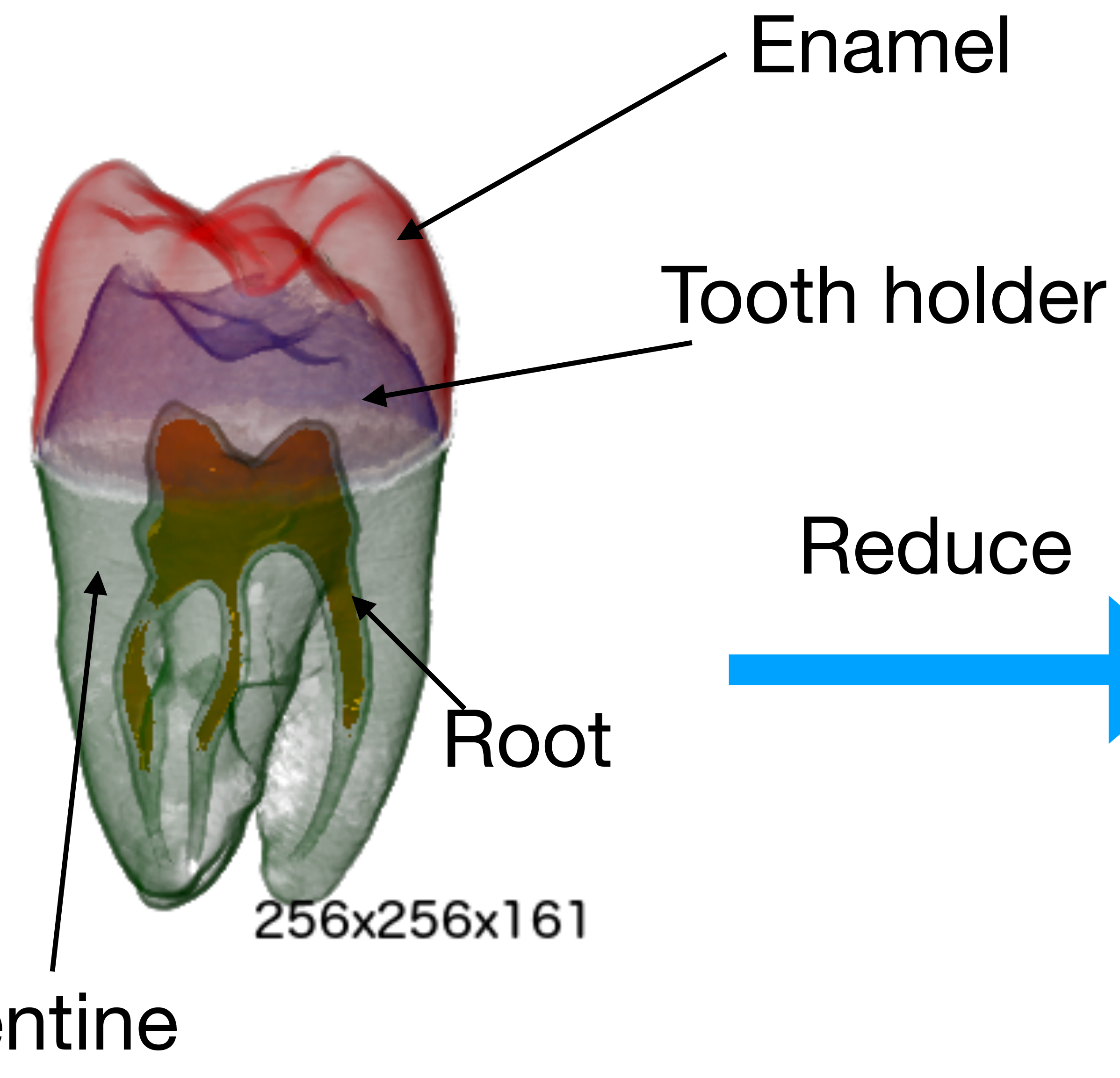
Osirix OBELIX dataset (http://medvis.org/datasets/)

T. M. Athawale, B. Ma, E. Sakhaee, L. Zhou, C. R. Johnson, and A. Entezari; **Nonparametric Models for Direct Volume Rendering of Uncertain Data Using Multidimensional Transfer Functions** (in progress)

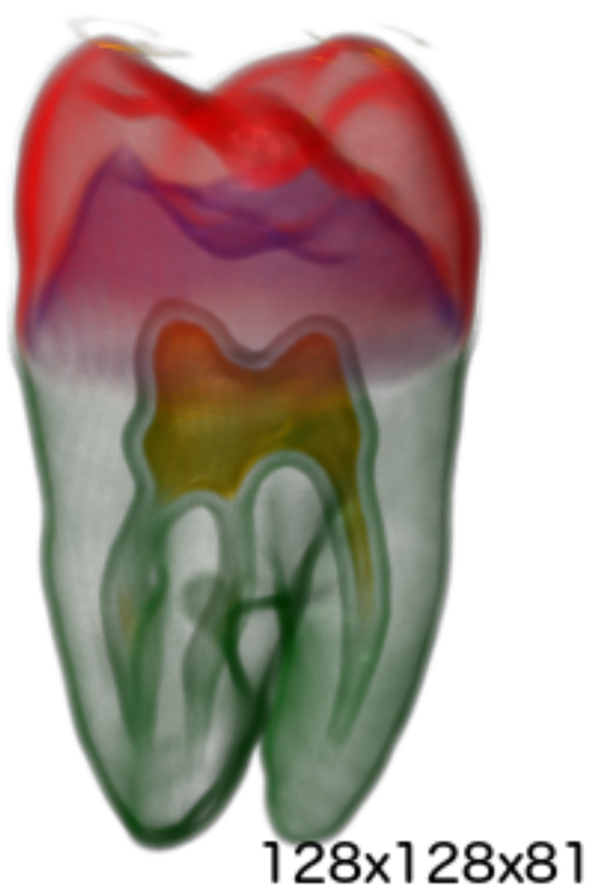
DVR of Uncertain Data (2D Transfer Functions)

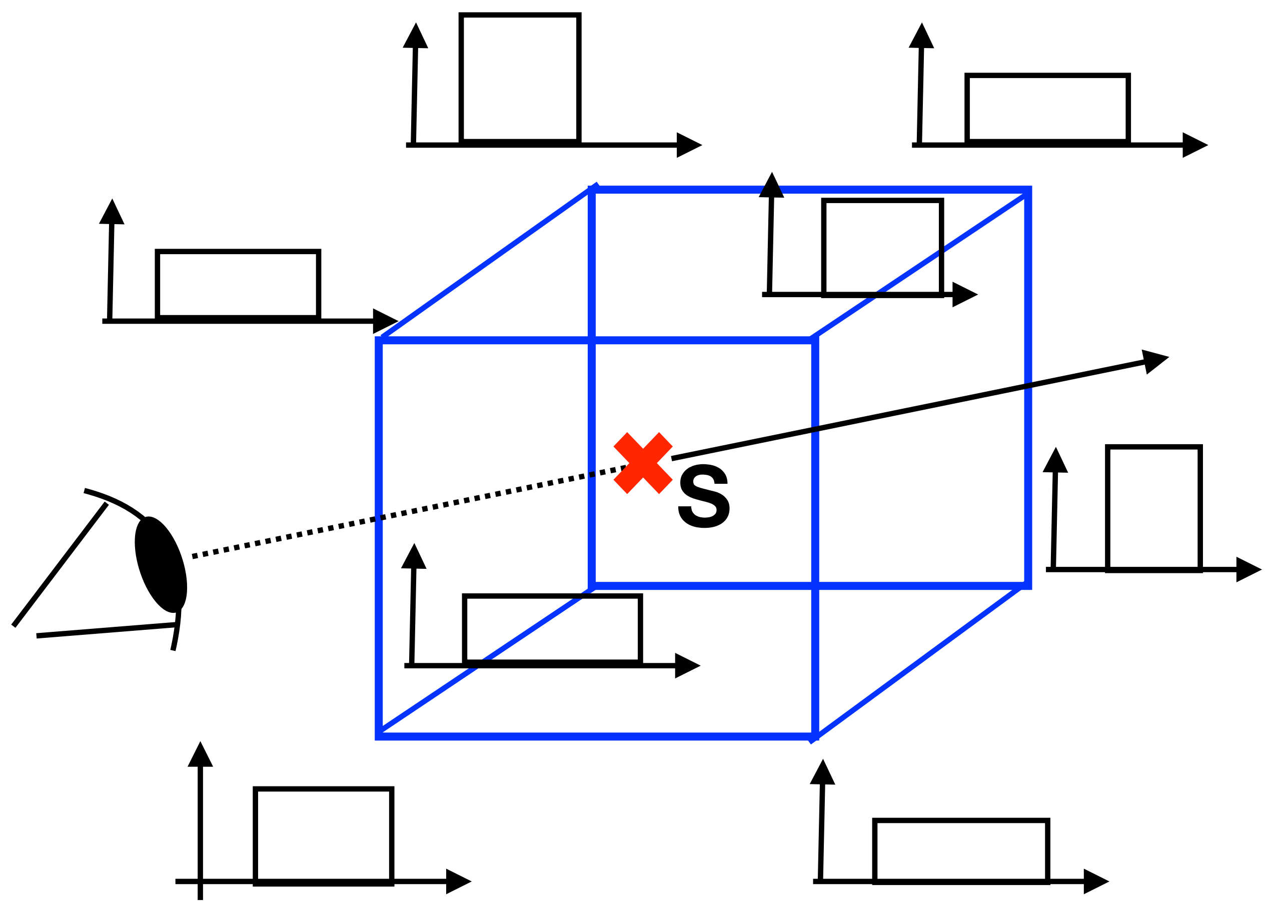
[Kniss et al., 2002]

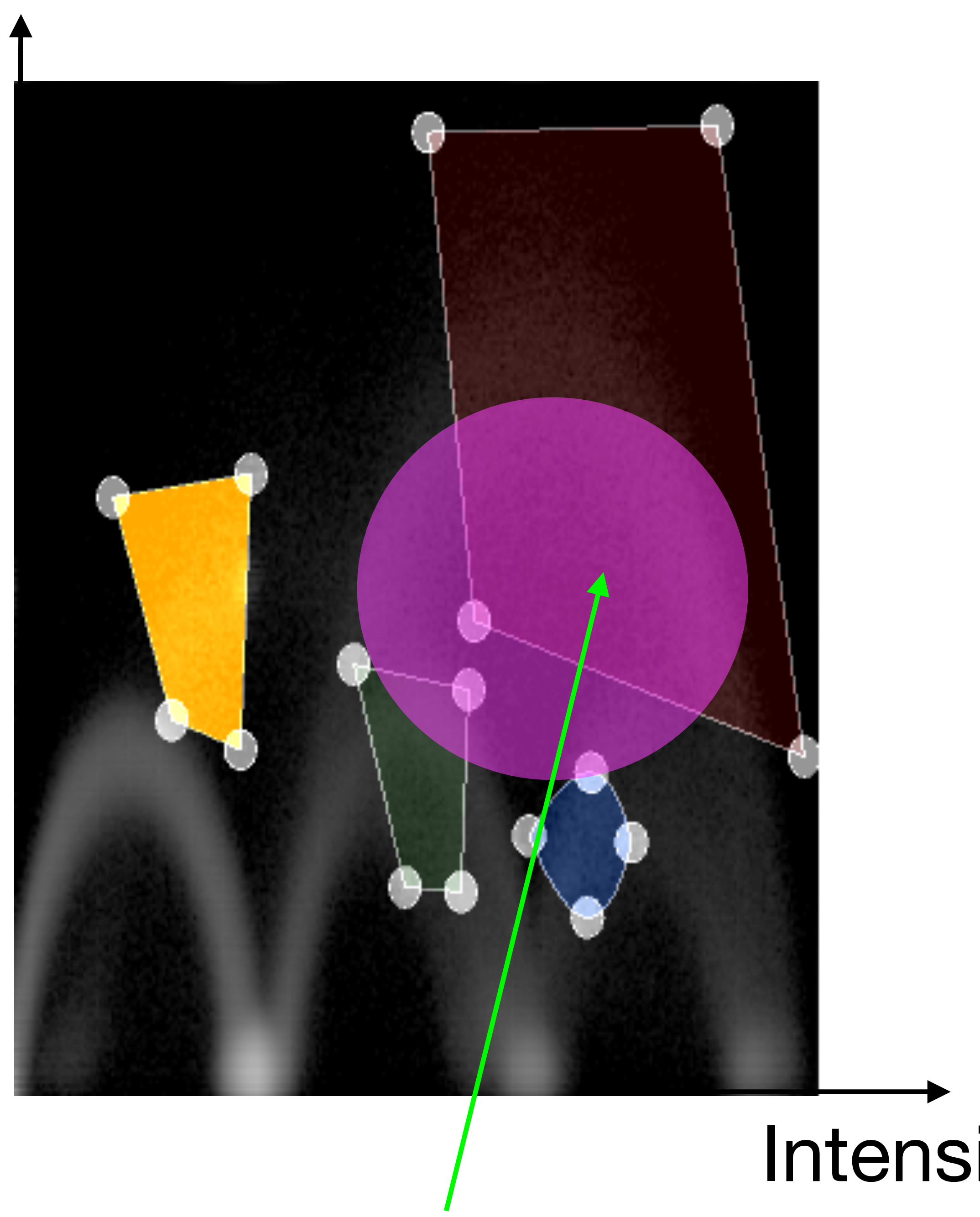




Reduce







Probability density of S:
The material for the sample
is most likely to be Enamel

A Statistical Framework for Visualization of Positional Uncertainty in Deep Brain Stimulation Electrodes

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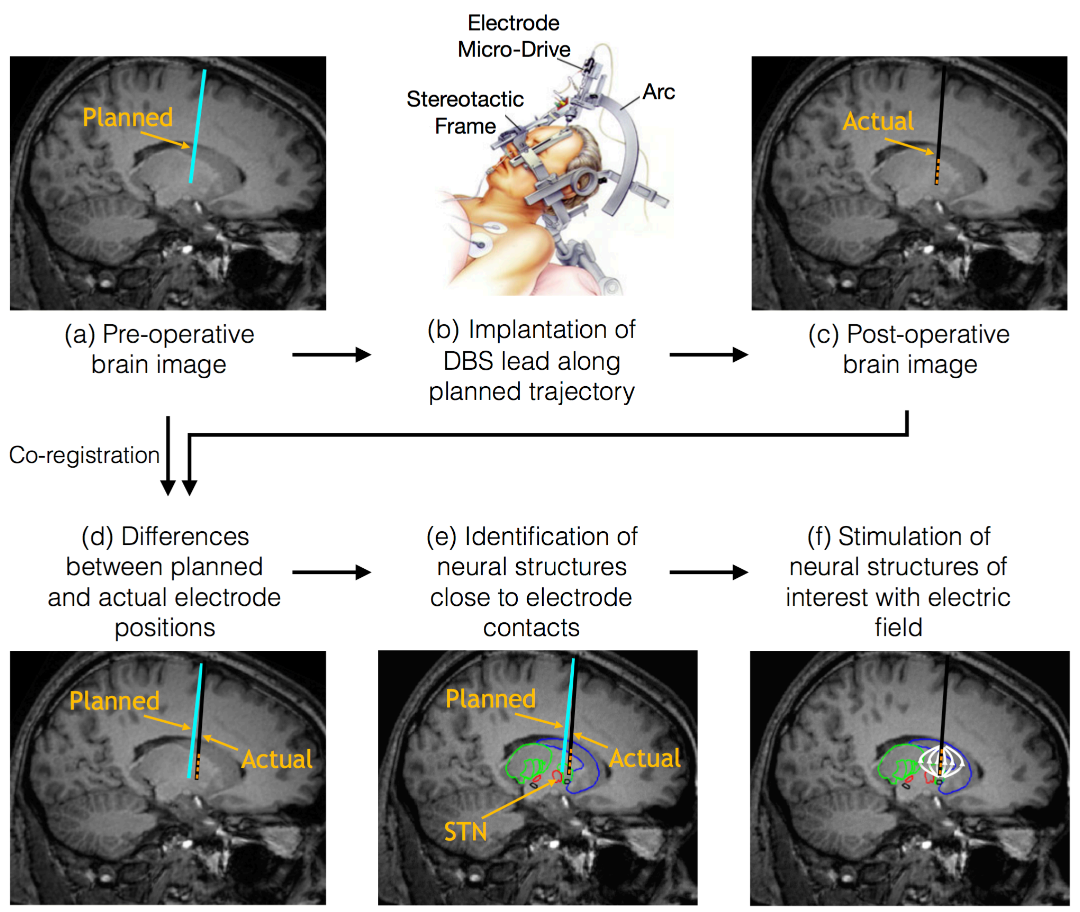
Deep Brain Stimulation (DBS)

- An FDA-approved neurosurgical procedure for treating patients with movement disorders, e.g., Parkinson's and dystonia.

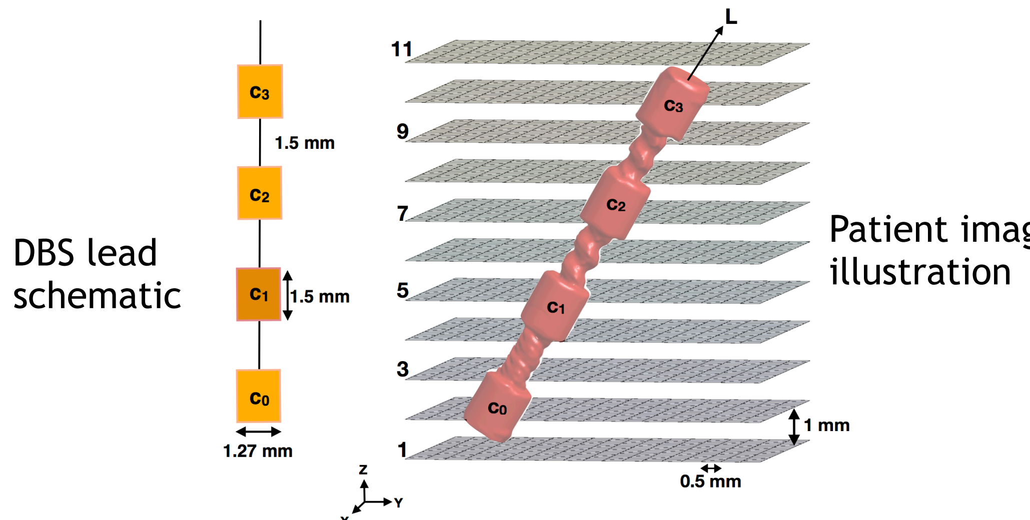


- The effectiveness of DBS depends upon physician's knowledge regarding precise *DBS electrode positions* in the patient brain.
- The role of post-operative DBS imaging:
  - Understand DBS electrode positions in the patient brain.
  - Mitigate the uncertainty in DBS electrode positions arising from mechanical inaccuracies of stereotactic frames [1] and brain shift [2].

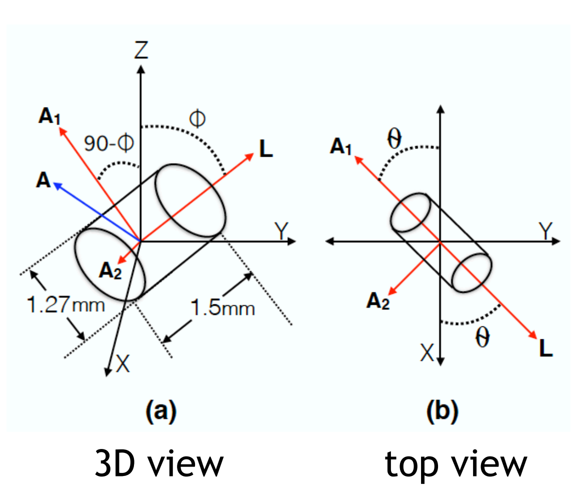
Problem with Post-Operative DBS imaging

- The finite resolution of post-operative imaging limits our knowledge of exact electrode positions in the patient brain.

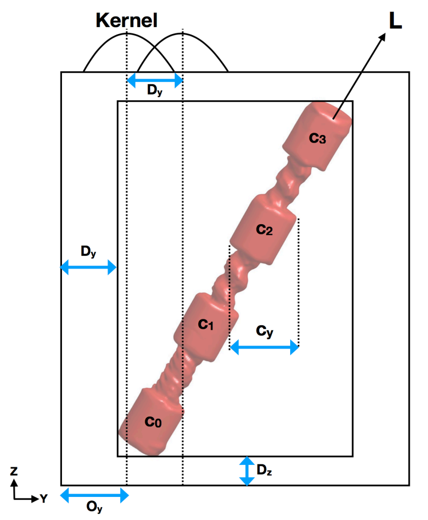
Post-Operative Imaging Uncertainty



- **Given:** A finite-resolution CT scan of implanted DBS electrodes, e.g., the image above captures data on 11 slices.
- **Goal:** To quantify the spatial uncertainty in DBS electrodes for their finite-resolution imaging and visualize the quantified spatial uncertainty.
- **Approach: (a)** Compute electrode geometry in closed form, **(b)** Map electrode geometry to a high-resolution electrode image (with electrodes out of patient brain), **(c)** Draw low-resolution samples from a high-resolution electrode image, **(d)** Compare low-resolution samples with the patient image [3].

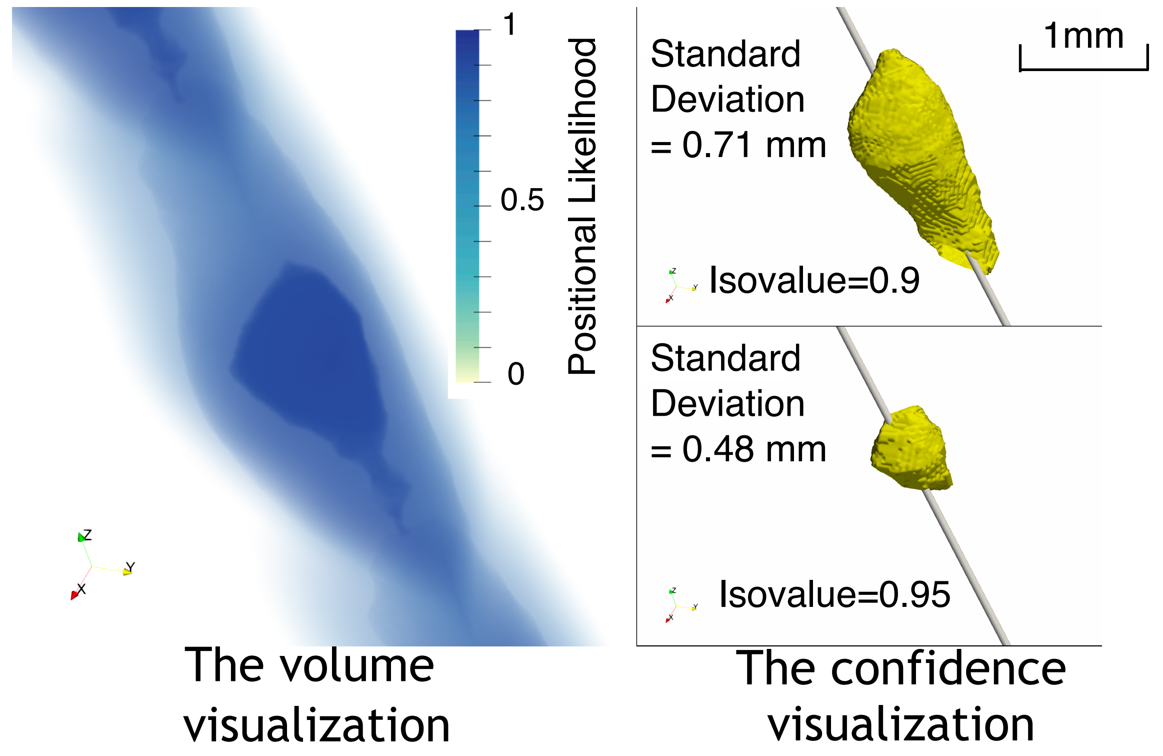


The DBS electrode geometry



Downsampling of the high-resolution image

Electrode-Center Spatial Uncertainty Visualizations



The volume visualization | The confidence visualization

Conclusion

We show that the uncertainty in DBS electrode positions is significant in post-operative imaging, e.g., 0.49 mm average spatial uncertainty for 0.45x 0.45x1 mm^3 resolution. Further, we integrate DBS computational modeling pipeline with our electrode uncertainty visualizations for accurate prediction of patient response to therapy.

Acknowledgements

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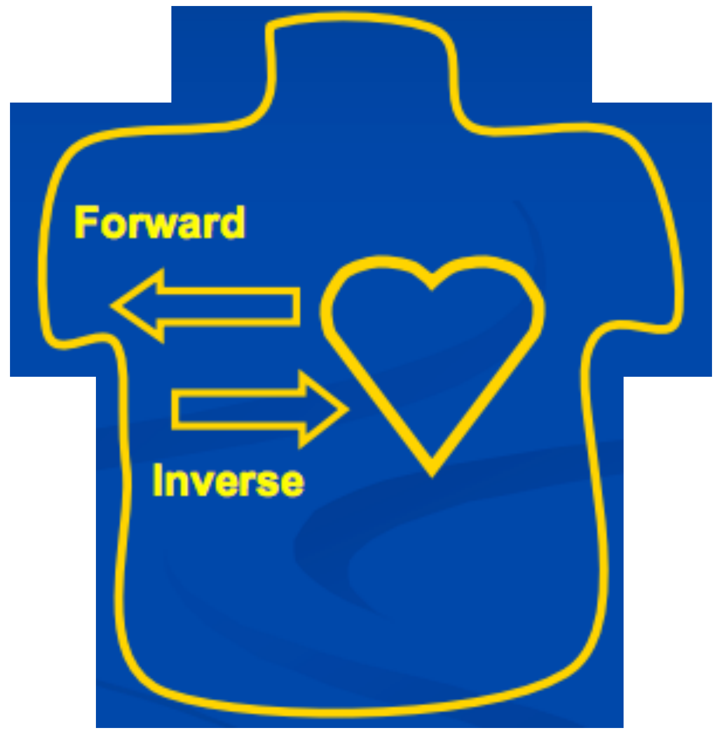
References

[1]: Maciunas RJ; Galloway RL, and Latimer JW. **The application accuracy of stereotactic frames.** *Neurosurgery, vol. 35, no. 4, pp. 682-694, 1994.*
[2]: Halpern CH., Danish SF, Baltuch GH, and Jaggi JL. **Brain shift during deep brain stimulation surgery for Parkinson's disease.** *Stereotact Funct Neurosurg, vol. 86, no. 1, pp. 37-43, 2008.*
[3]: Athawale T, Johnson K, Butson CR, and Johnson CR. **A statistical framework for visualization of positional uncertainty in deep brain stimulation electrodes.** *Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization, vol.7, no. 4, pp. 438-449, 2019.*

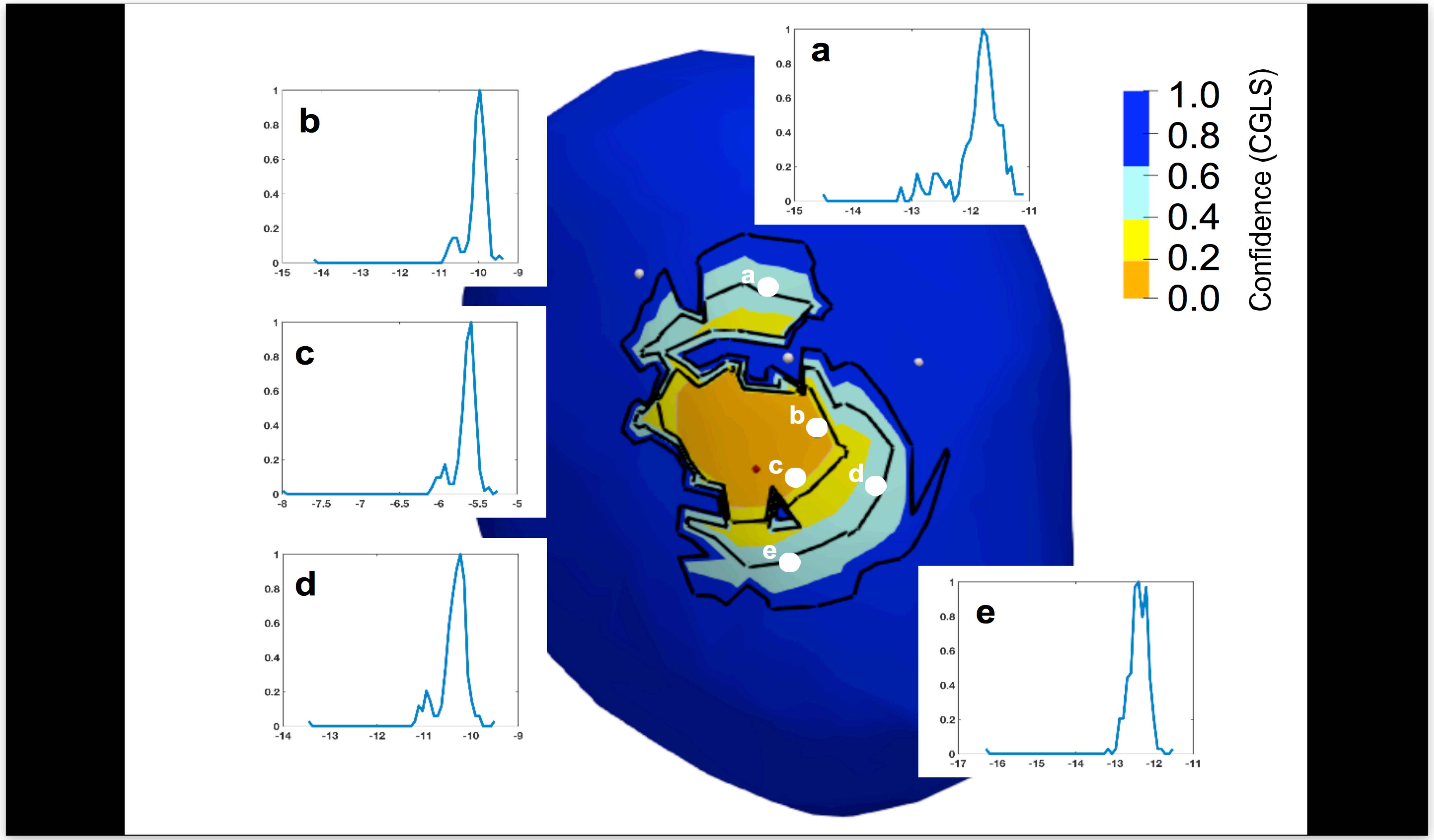
Uncertainty Visualization for Domain-Specific Data

Positional uncertainty in sources of arrhythmia for noisy ECG recordings

ECGI



Ensemble data representing the uncertainty in cardiac potentials



Positional Likelihood of Sources of Arrhythmia

T. M. Athawale, D. Njeru, J. France, and C. R. Johnson; **Quantifying and Visualizing Uncertainty for Source Localization in Electrocardiographic Imaging** (in progress)

Conclusions

- Uncertainty visualizations are important for avoiding misleading interpretations regarding the underlying data
  - Level sets
  - Deep brain stimulation imaging
  - Morse complex visualizations
  - Direct volume rendering
  - Electrocardiography imaging
- Statistical methods for uncertainty quantification
  - Monte Carlo vs. Analytical
- Methods for uncertainty visualization
  - Color mapping proportional to the level of confidence/uncertainty
  - Interactive probability distribution queries
  - Derive uncertainty volumes and visualize them using isosurfaces/direct volume rendering

Future Work

- Visualization algorithms, e.g., topological analysis, for uncertain input data
- Uncertainty visualization for domain-specific data
- Machine learning and uncertain data
- Value of uncertainty visualizations (are they informative or confusing to a user?)

Publications

- T. M. Athawale and C. R. Johnson; **Probabilistic Asymptotic Decider for Topological Ambiguity Resolution in Level-Set Extraction for Uncertain 2D Data**, *IEEE Transactions on Visualization and Computer Graphics (TVCG), Special Issue on IEEE VIS Conf*, vol.25, no. 1, pp. 1163-1172, Jan. 2019.

- T. M. Athawale, K. A. Johnson, C. R. Butson, and C. R. Johnson; **A Statistical Framework for Visualization of Positional Uncertainty in Deep Brain Stimulation Electrodes.**, *Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization*, pp. 1-12, Oct. 2018.

- T. M. Athawale, E. Sakhaee, and, A. Entezari; **Isosurface Visualization of Data with Nonparametric Models for Uncertainty**, *IEEE Transactions on Visualization and Computer Graphics (TVCG), Special Issue on IEEE VIS Conf*, vol.22, no.1, pp.777-786, Jan. 2016.

- T. M. Athawale and A. Entezari.; **Uncertainty Quantification in Linear Interpolation for Isosurface Extraction**, *IEEE Transactions on Visualization and Computer Graphics (TVCG), Special Issue on IEEE VIS Conf*, vol.19, no.12, pp.2723-2732, Dec. 2013.

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(Direct volume rendering project)

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