VIS2021

Uncertainty Visualization of the Marching Squares and Marching Cubes Topology Cases

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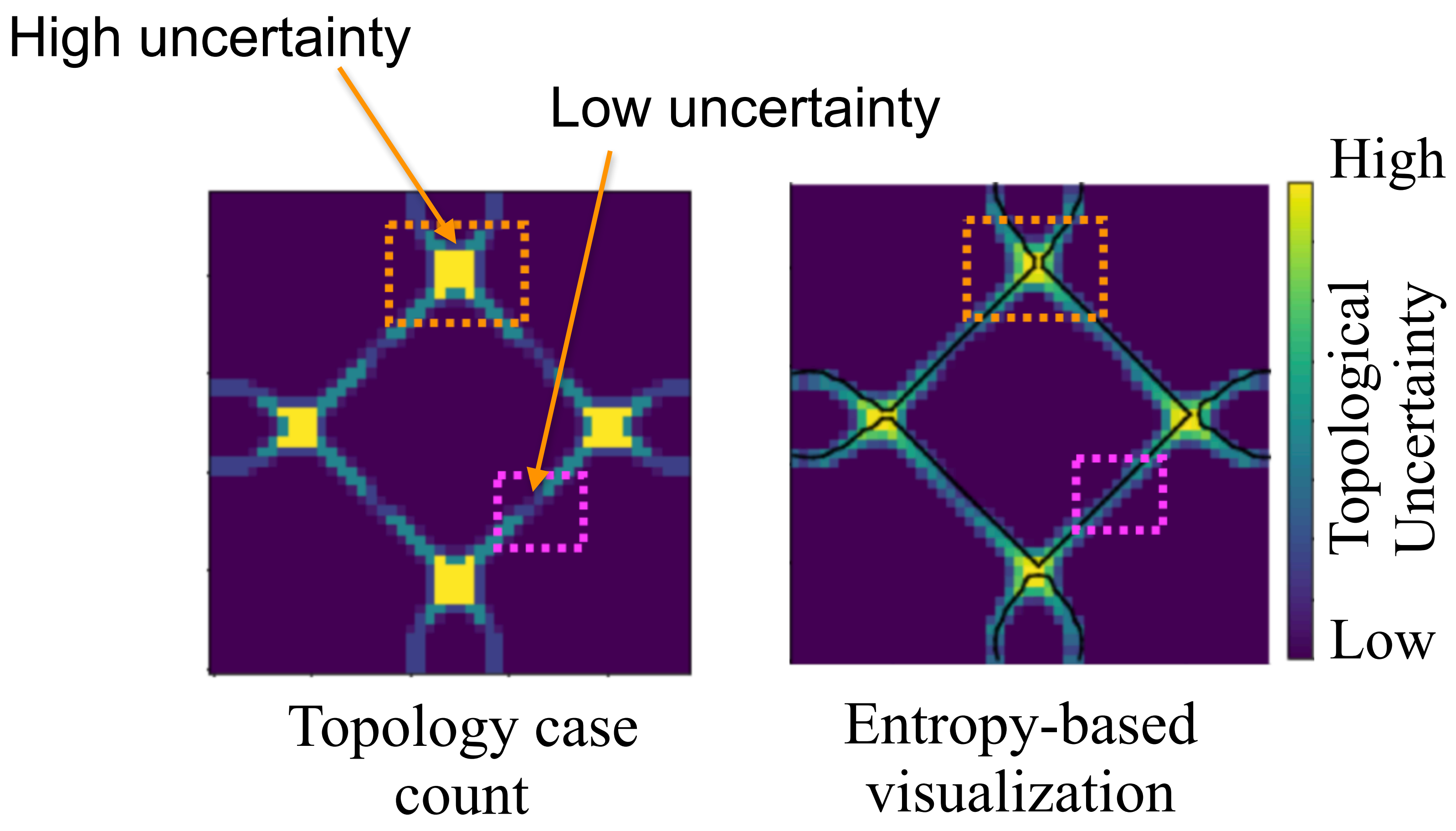
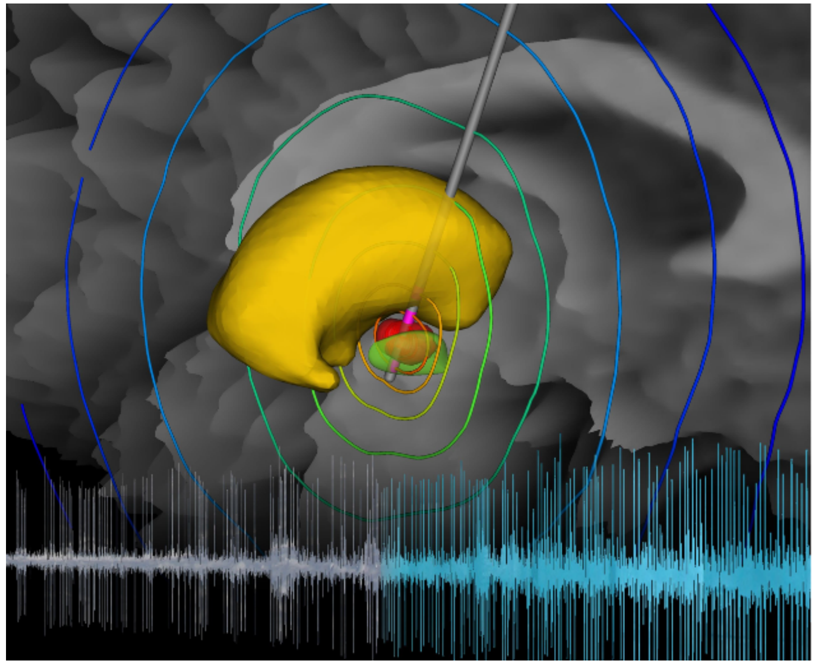
Outline

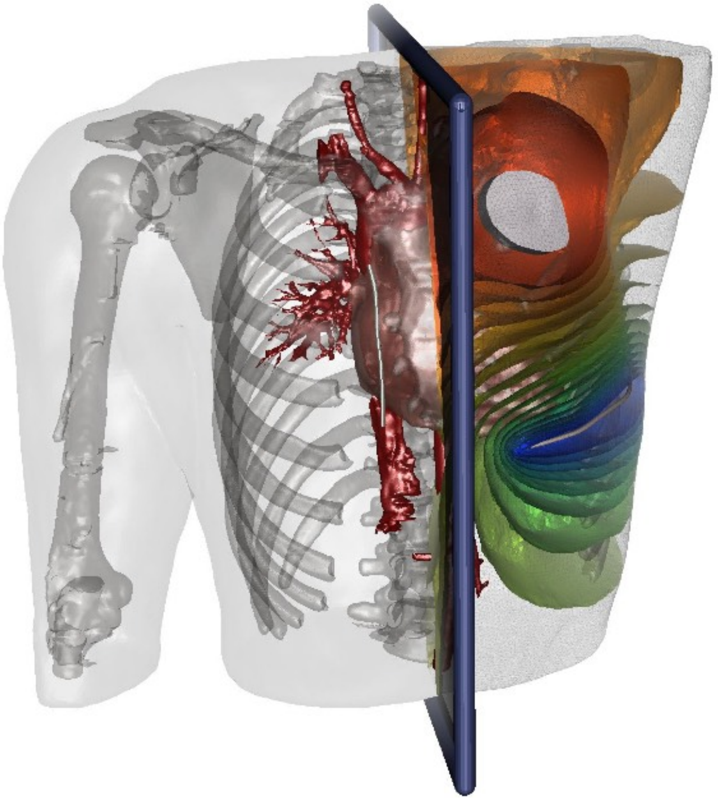
- Marching squares/cubes algorithm [Lorensen and Cline, 1987]
  - Topology cases for fixed/certain data
- Research question
  - Topology cases in noisy/uncertain data
  - Relevant prior work
- Our topological uncertainty visualization framework
  - Topology case count
  - Entropy-based approach
- Results, conclusion, and future work



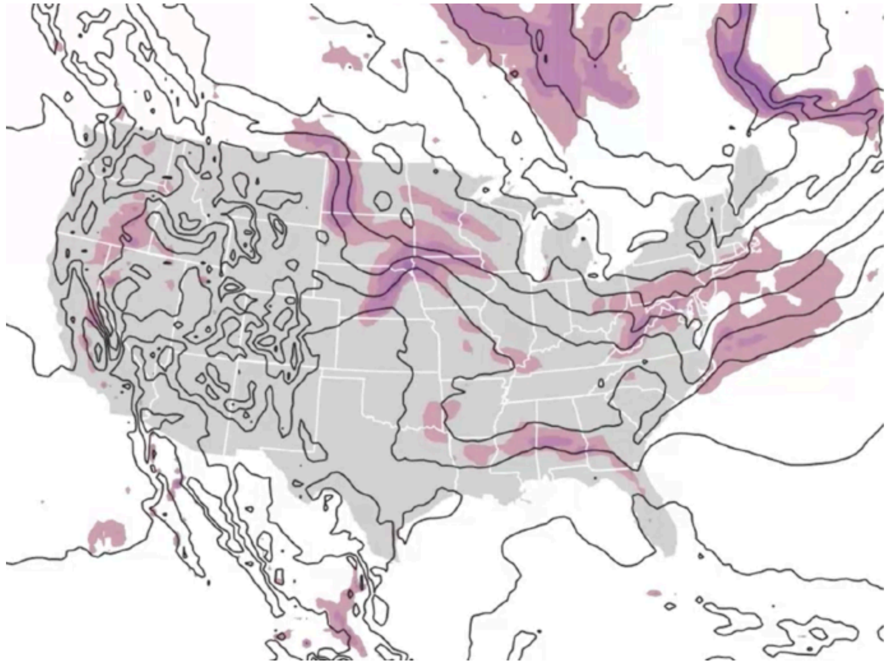
Level-set Visualizations



Deep Brain Stimulation (DBS)



Bioelectric-field Simulation

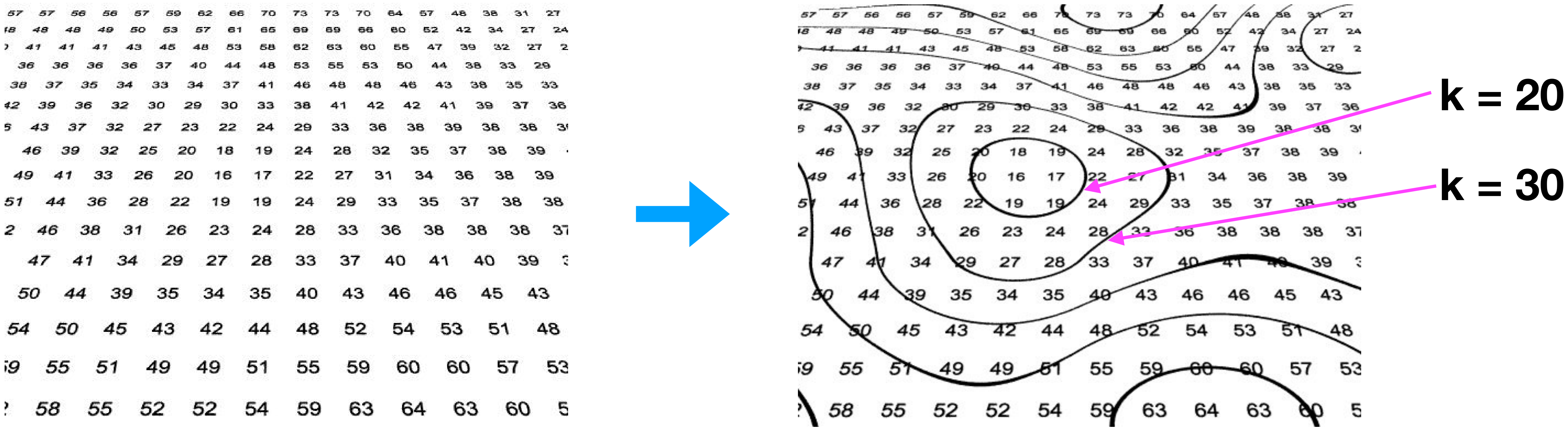


Temperature Field

Level-set Extraction

The inverse problem: The level-set *S* corresponding to the isovalue *k* is given by:

$$S = \{x \in \mathbb{R}^n | f(x) = k\}$$, where $f : \mathbb{R}^n \rightarrow \mathbb{R}$



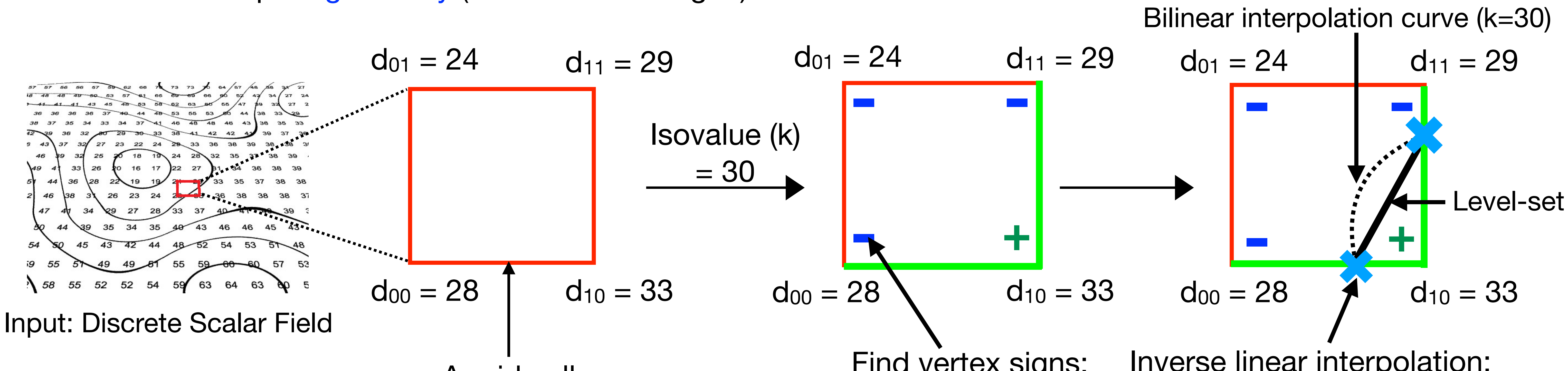
Input: Discrete Scalar Field

Output: Level-Sets Visualization

Marching Squares Algorithm [Lorensen and Cline, 1987]

For each grid cell (assume bilinear interpolation model):

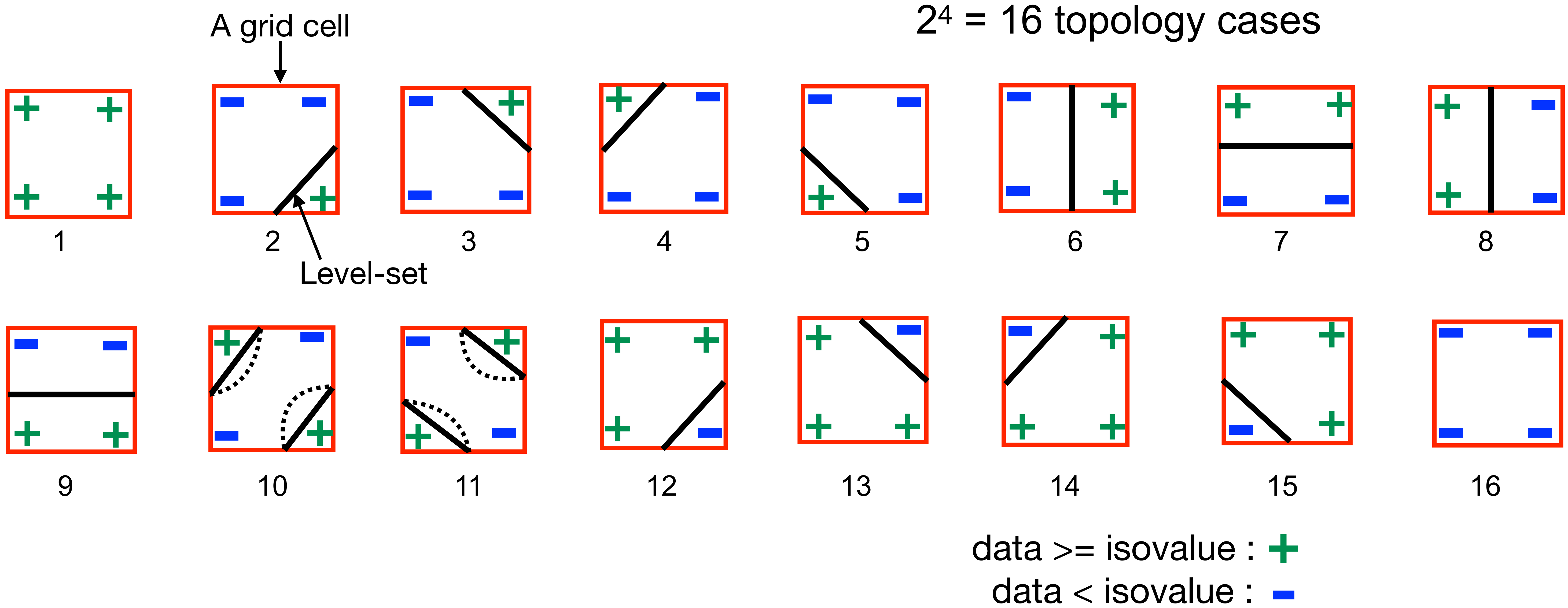
- Extract level-set topology (Which edges are crossed?)
- Compute geometry (Where on the edge?)



$$\alpha = \frac{k - d_{00}}{d_{10} - d_{00}} = \frac{30 - 28}{33 - 30} = 0.66$$

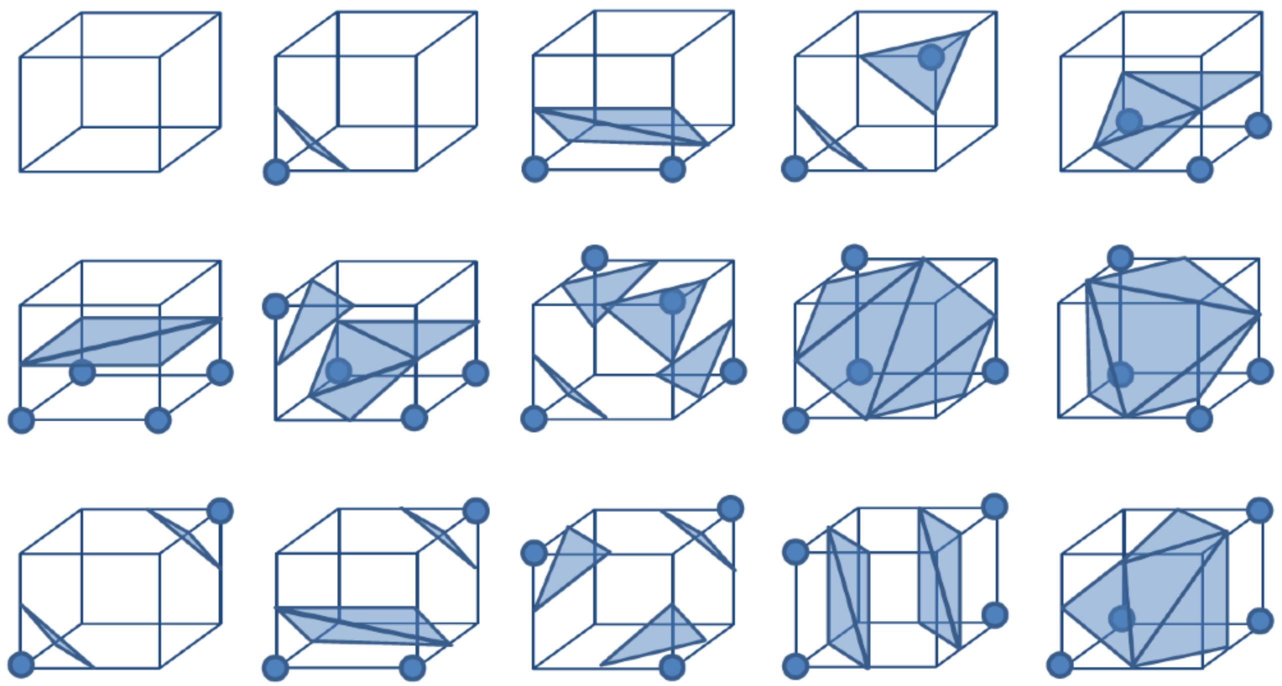
$$\alpha \in [0, 1]$$

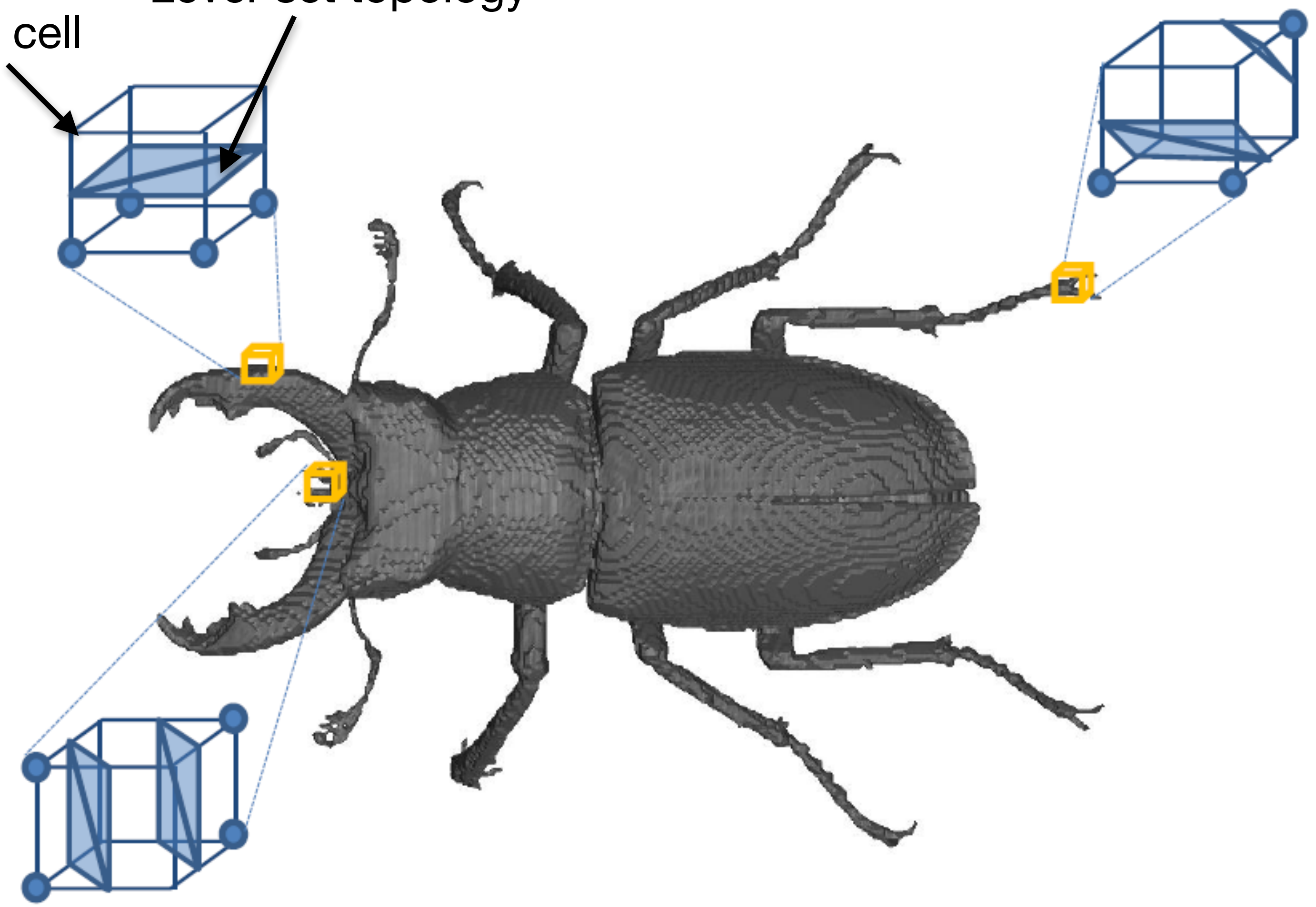
Marching Squares Algorithm: Topology Cases



Marching Cubes Algorithm: Topology Cases

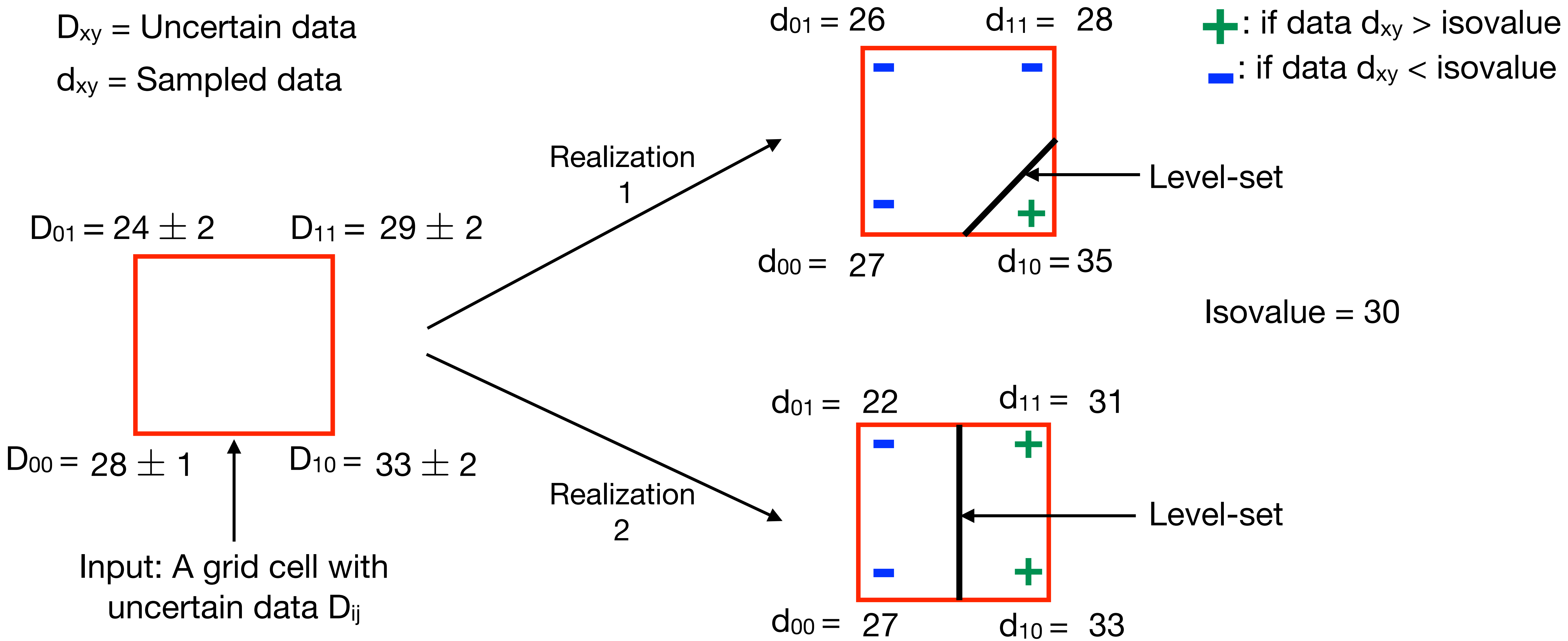
2^8 = 256 topology cases





The Stag Beetle dataset is courtesy of Vienna University of Technology
https://www.cg.tuwien.ac.at/research/vis/datasets/

Research Question: Marching Squares/Cubes Topology for Uncertain Data

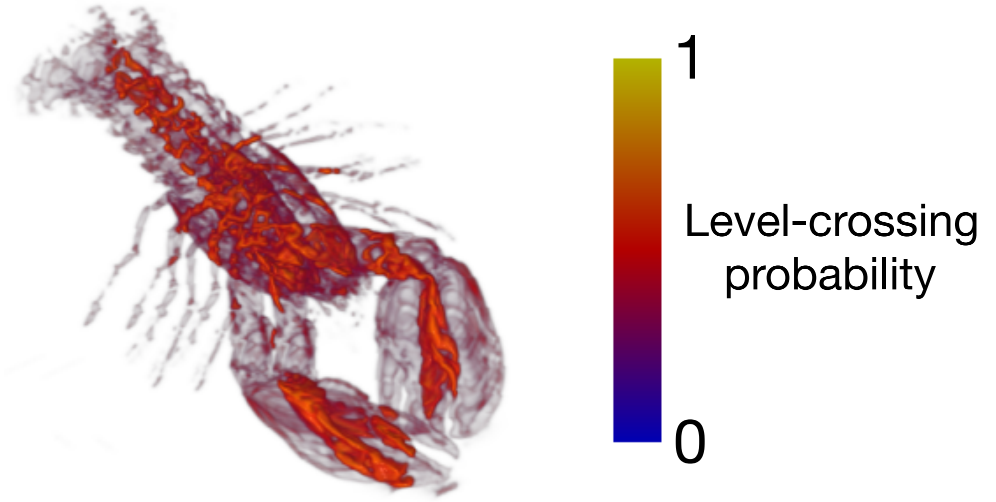


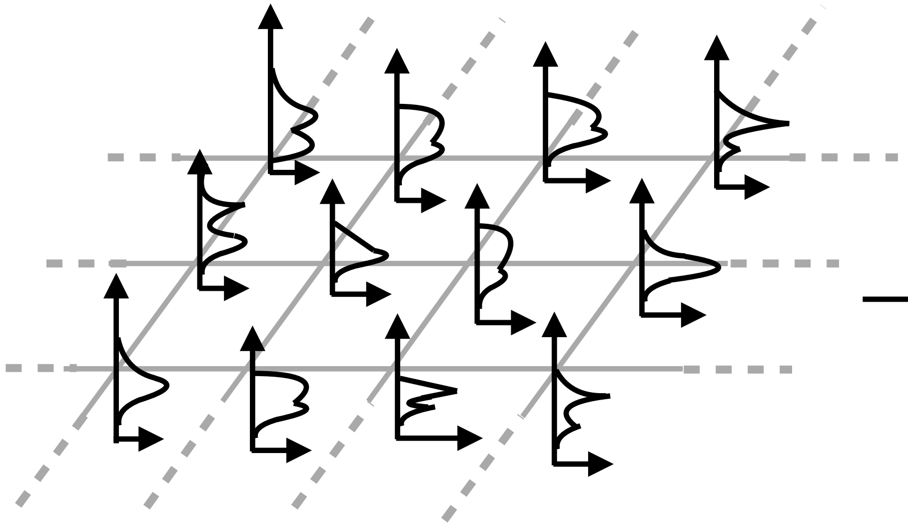
Research Question: Marching Squares/Cubes for Distribution Fields

Uncertainty visualization: the top research challenge. ***[Johnson and Anderson, 2004]***

Compute probability of level-set crossing per grid cell

(Probabilistic marching cubes [Pöthkow et al., 2011])



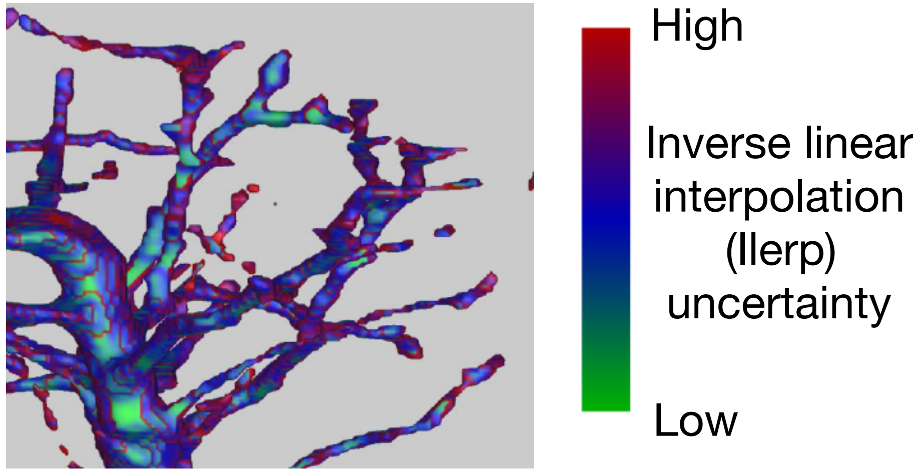


Input: Distribution field

([Thompson et al., 2011], [Wang et al., 2018])

Compute positional uncertainty of level-set crossing on grid cell edges

([Athawale et al., 2013])



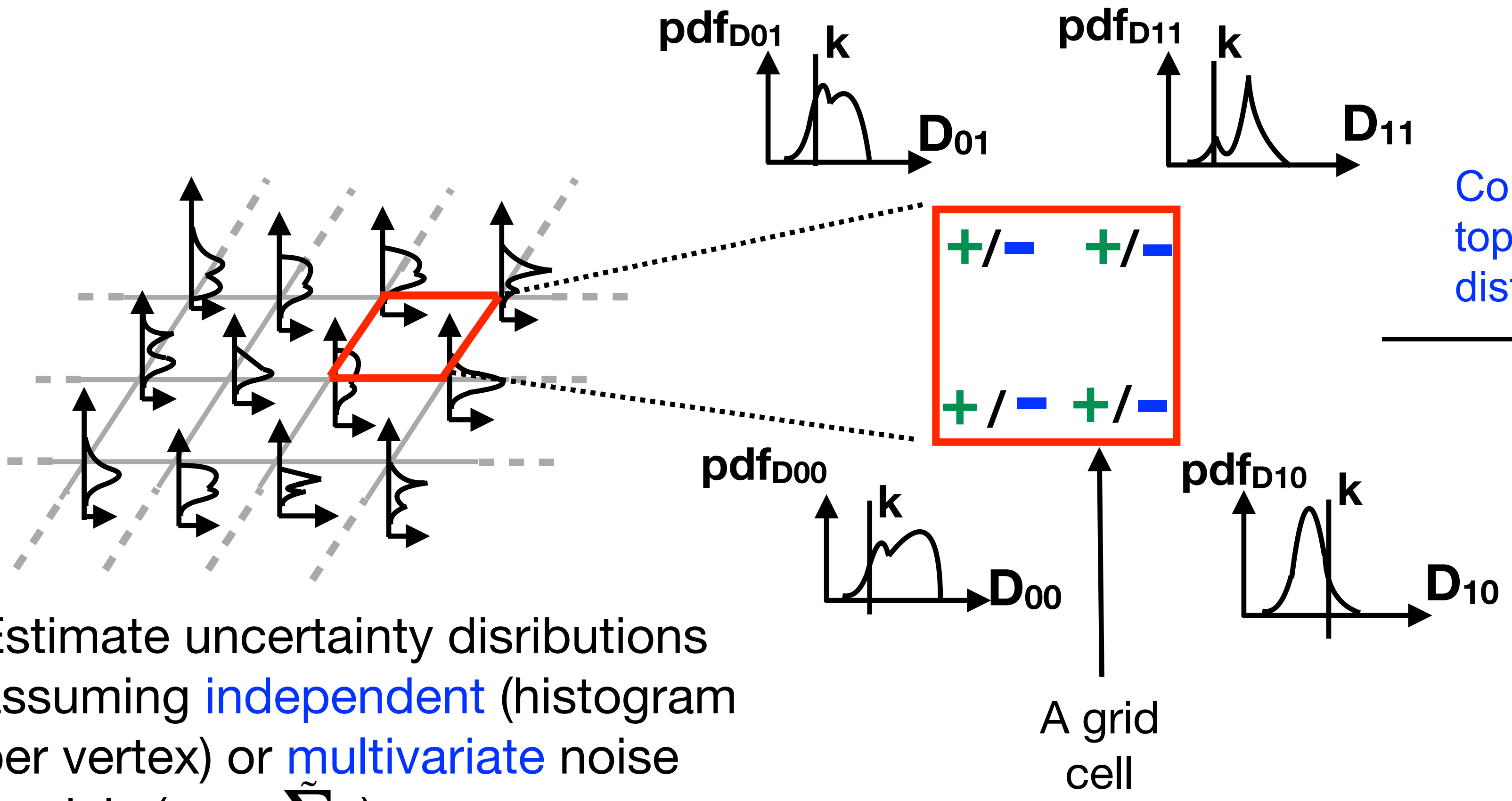
Our contribution

Uncertainty Quantification and Visualization of the marching cubes/squares topology cases

Our Framework for Uncertainty Visualization of Level-sets

Compute Probability Distribution of the Marching Squares/Cubes Topology Cases

k = Isovalue D_{xy} = Uncertain data $pdf_{D_{xy}}$ = Probability distribution of D_{xy}



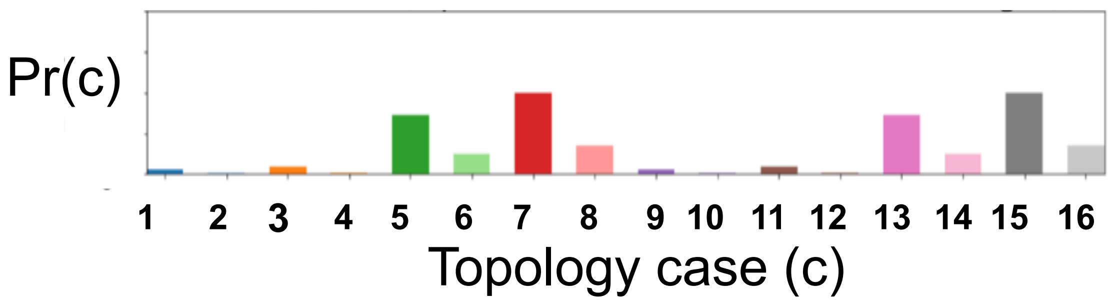
Estimate uncertainty disributions assuming independent (histogram per vertex) or multivariate noise models ($\mathcal{N}(\tilde{\mu}, \sum)$)

Compute topological distribution →

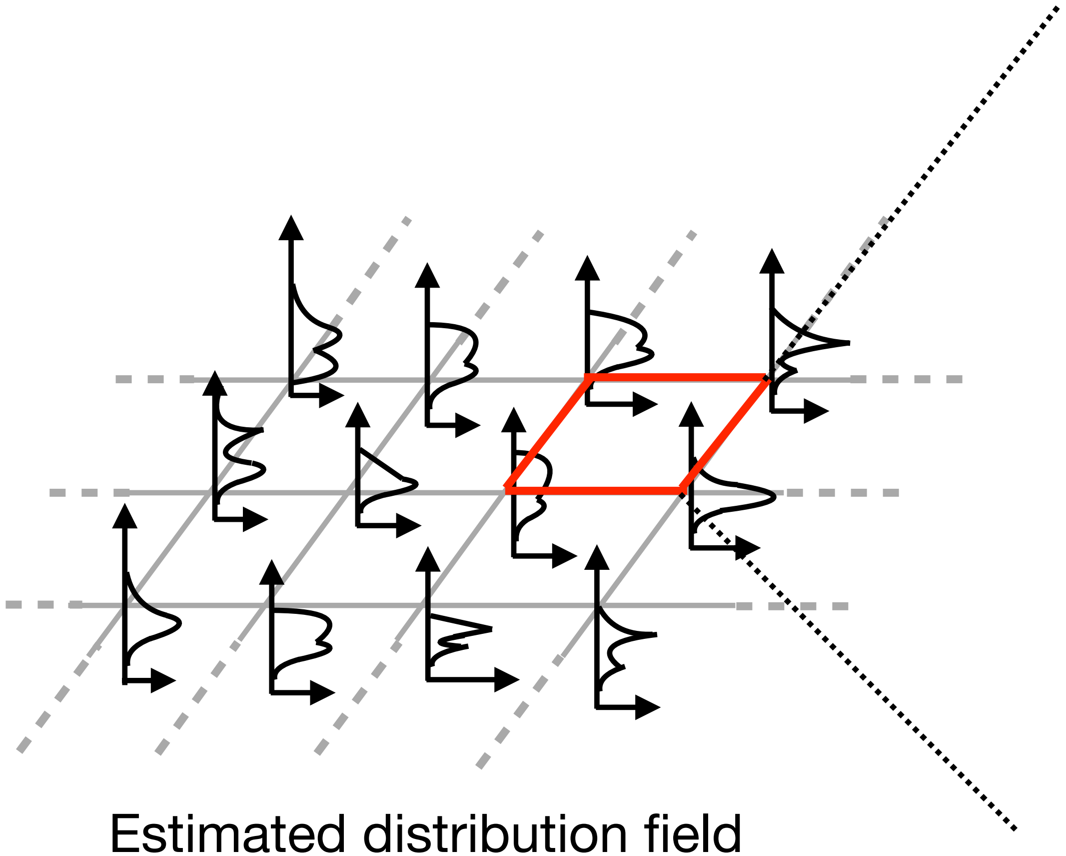
Monte Carlo approach:

1) Draw N samples from estimated distributions

2) Derive a histogram of topology cases based on the topology case observed for each of the N samples

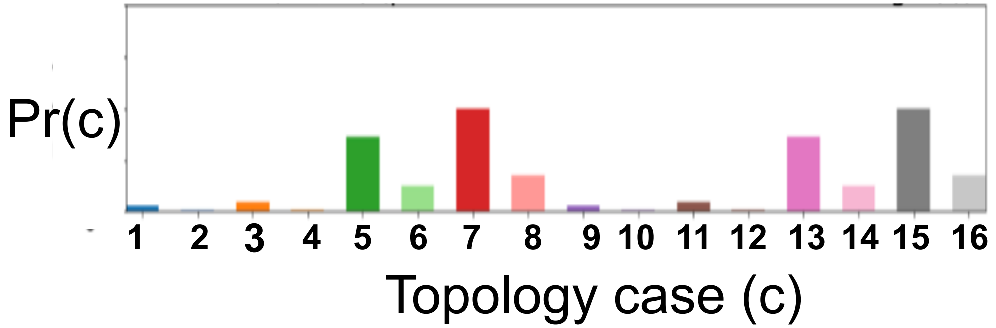


Topology Case Count Technique

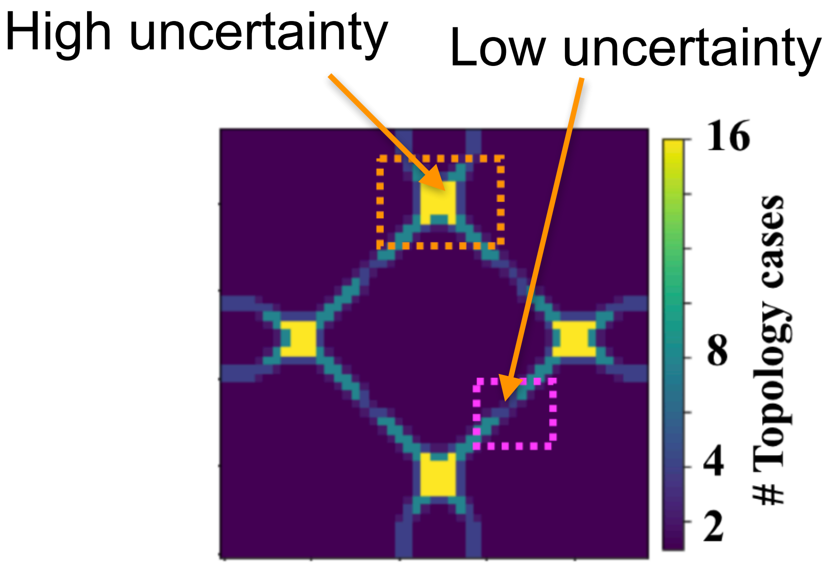


Estimated distribution field

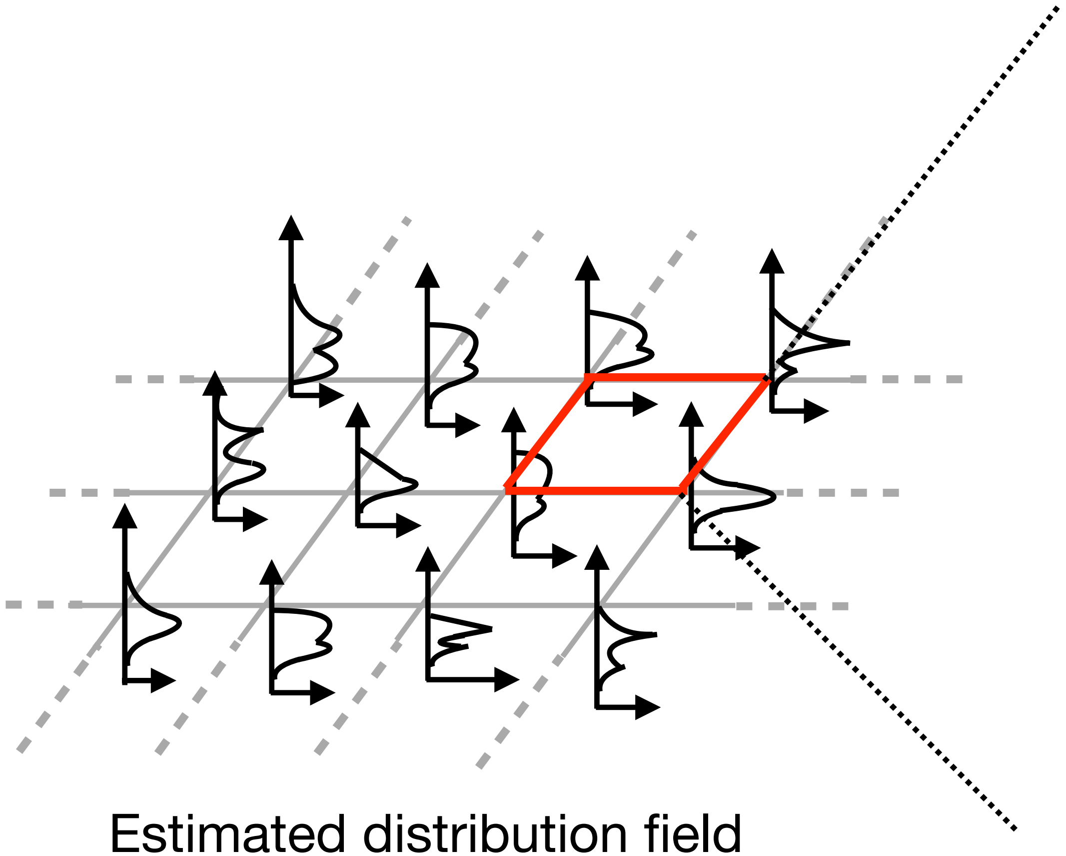
1) Compute probability distribution of the marching squares/cubes topology cases per grid cell.



2) Count and visualize the number of the topology cases with nonzero probability, e.g., in the example above there are 11 topology cases with nonzero probablity.



Entropy-based Uncertainty Quantification



Estimated distribution field

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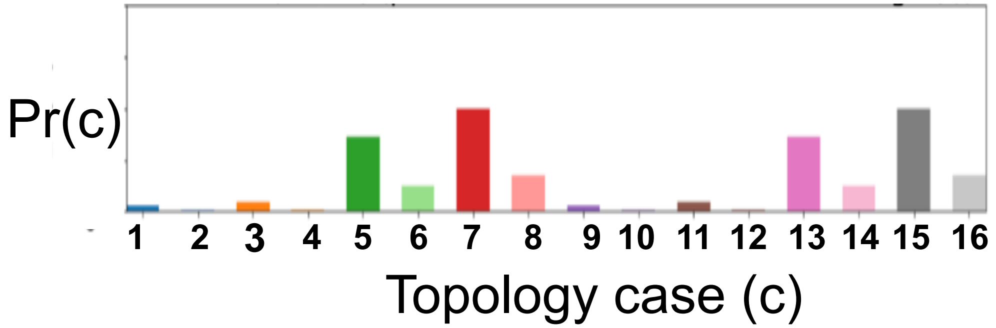
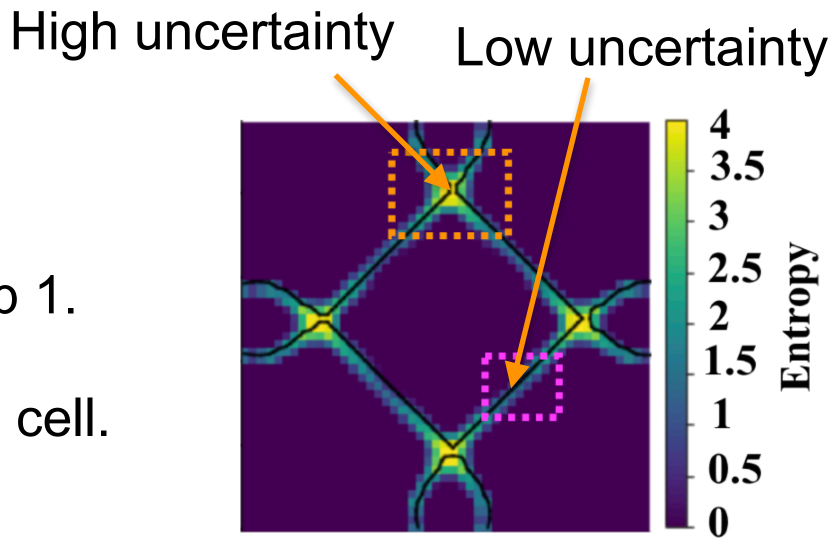


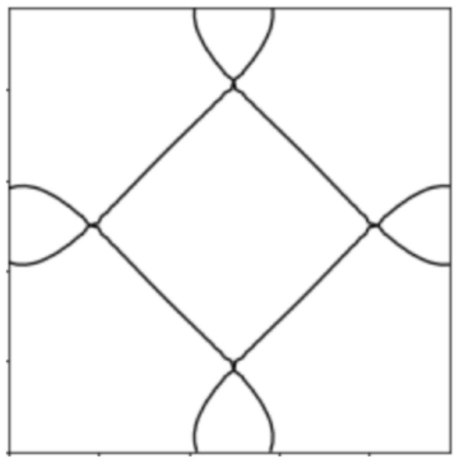
2) Compute and visualize entropy of the probability distribution computed in step 1. The high entropy implies relatively high randomness of topology cases within a cell.

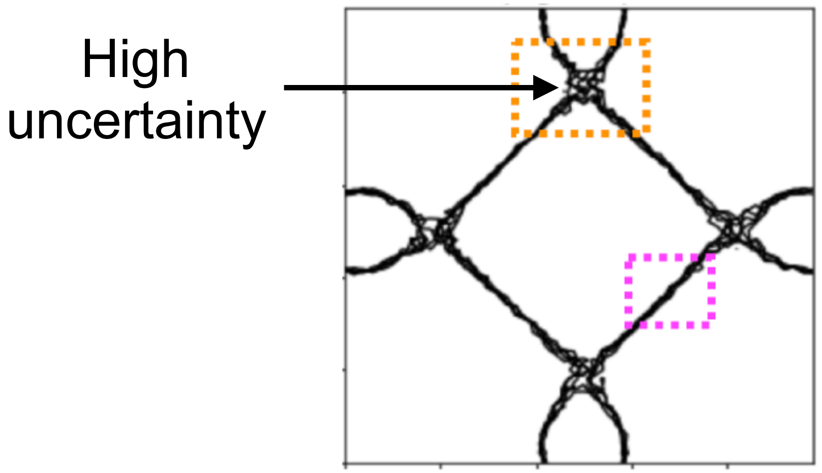


Results

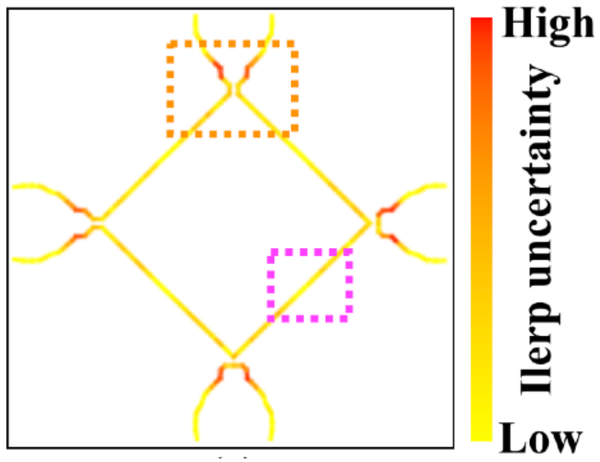
Synthetic Experiment: Ackley Dataset [Ackley, 1987]

- Independent uniform noise assumption
- Uncertainty visualizations for an ensemble with 50 members

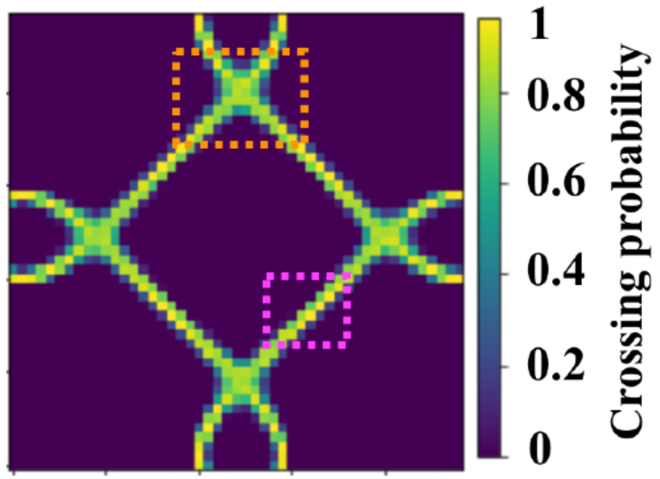

(a) Level-set in an image without noise



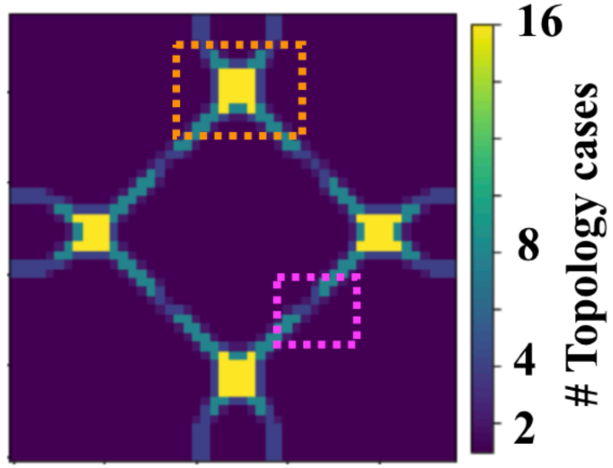
(b) Spaghetti plot [Potter et al., 2009]



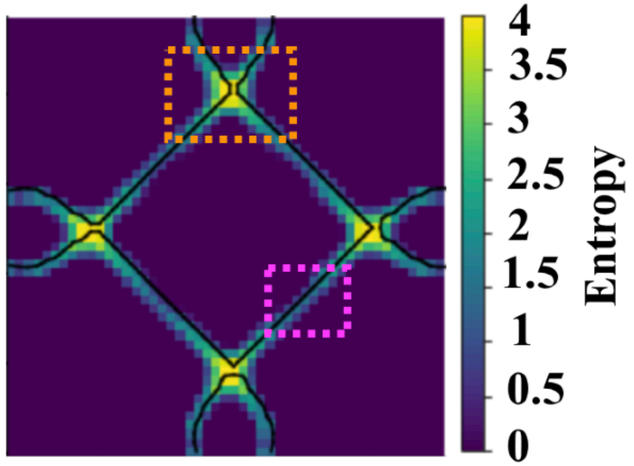
(c) Ilerp uncertainty [Athawale et al., 2013]



(d) Probabilistic marching squares [Pöthkow et al., 2011]



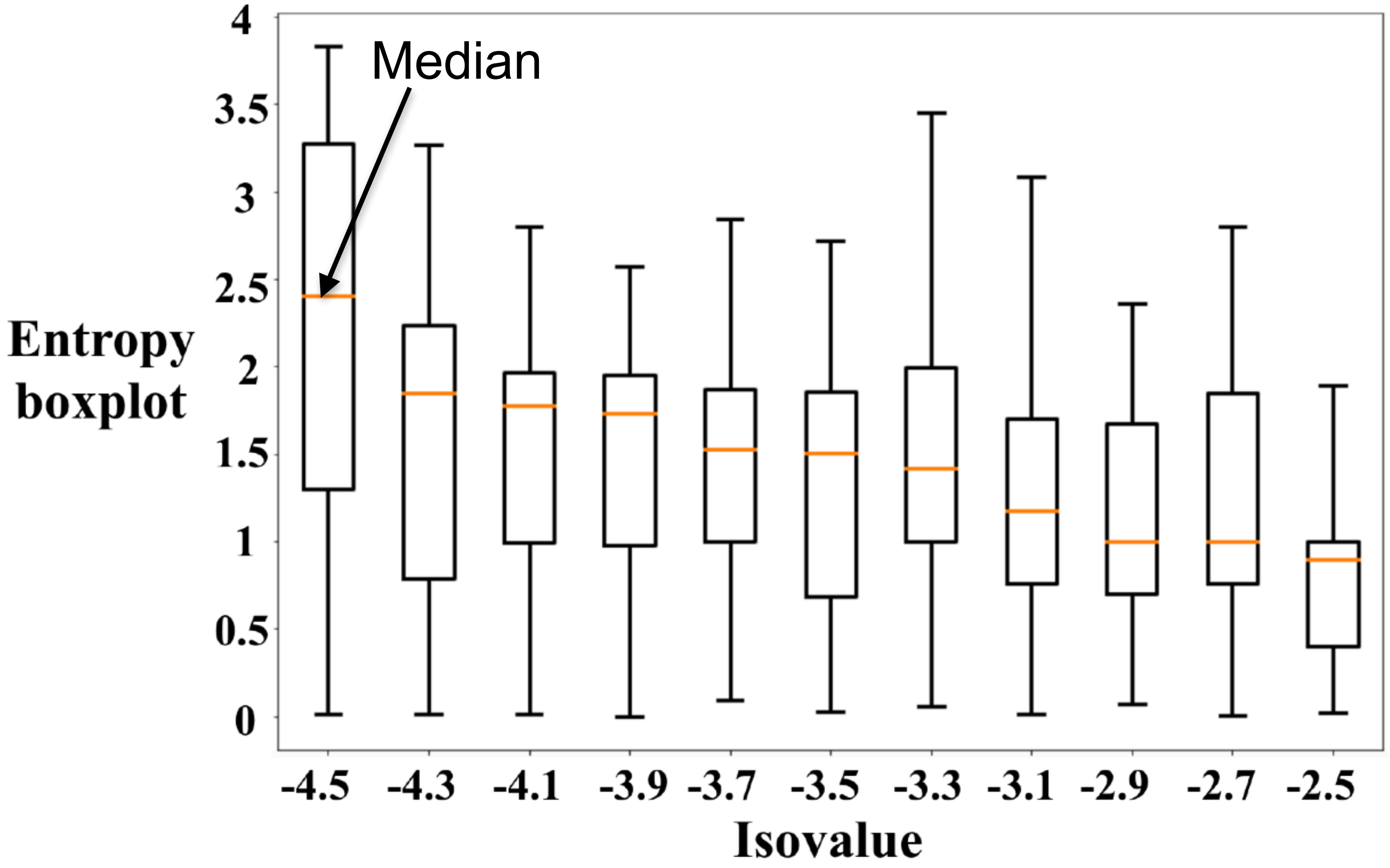
(e) Topology case count (our contribution)



(f) Entropy-based uncertainty quantification (our contribution)

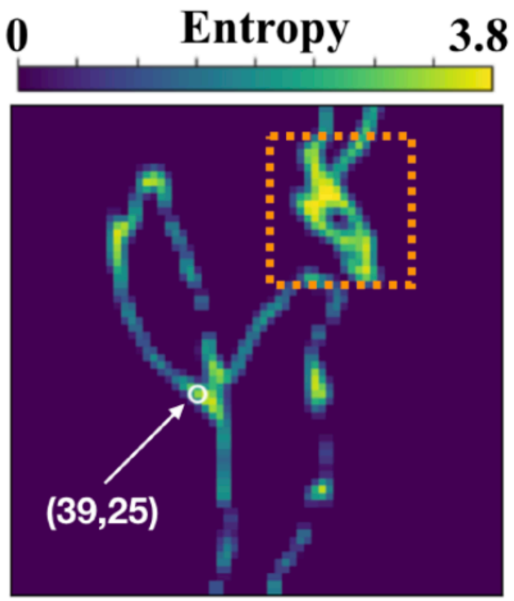
Uncertainty Distribution of Level-sets: Ackley Dataset

The median entropy for the isovalue -4.5 is greater than the one for the isovalue -2.9

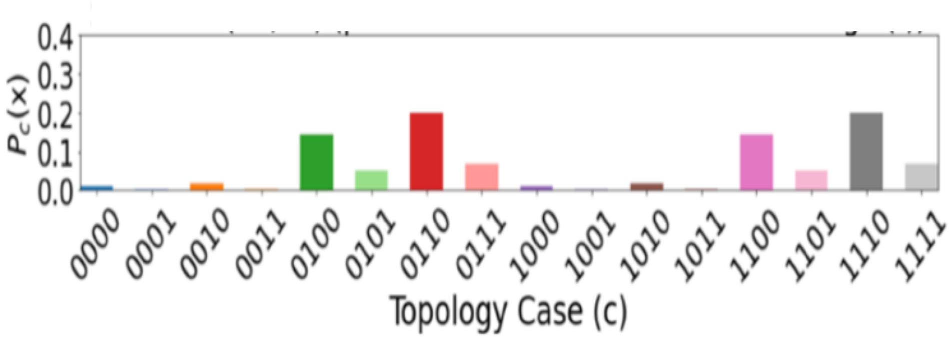


Wind Dataset: Multivariate Vs. Independent Noise

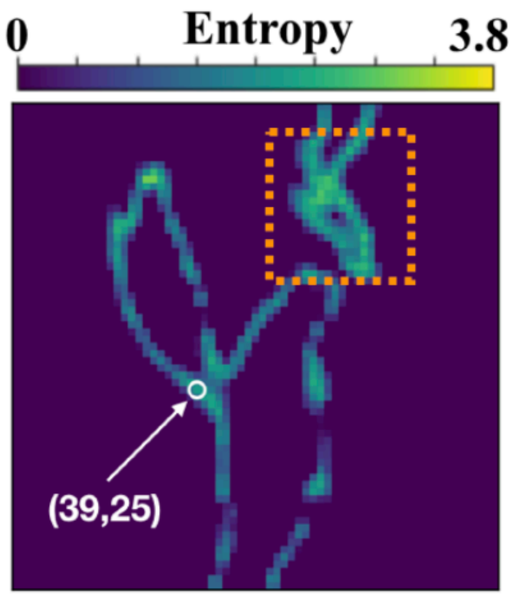
Interactive probability queries [Potter et al., 2012]



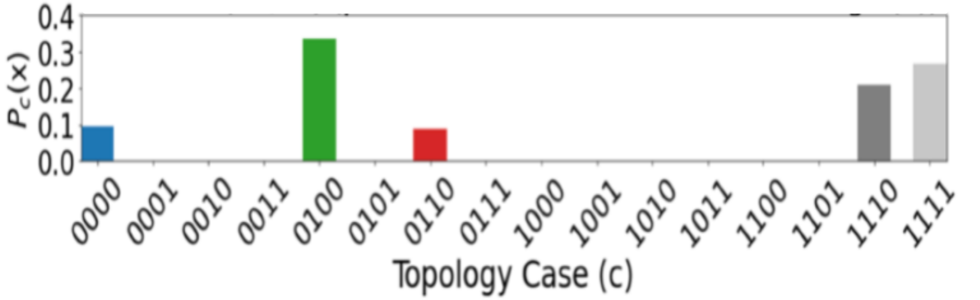
(a) Entropy-based visualization (independent noise)



(b) Topology distribution for pixel (39,25) of image (a) (independent noise)



(c) Entropy-based visualization (multivariate noise)

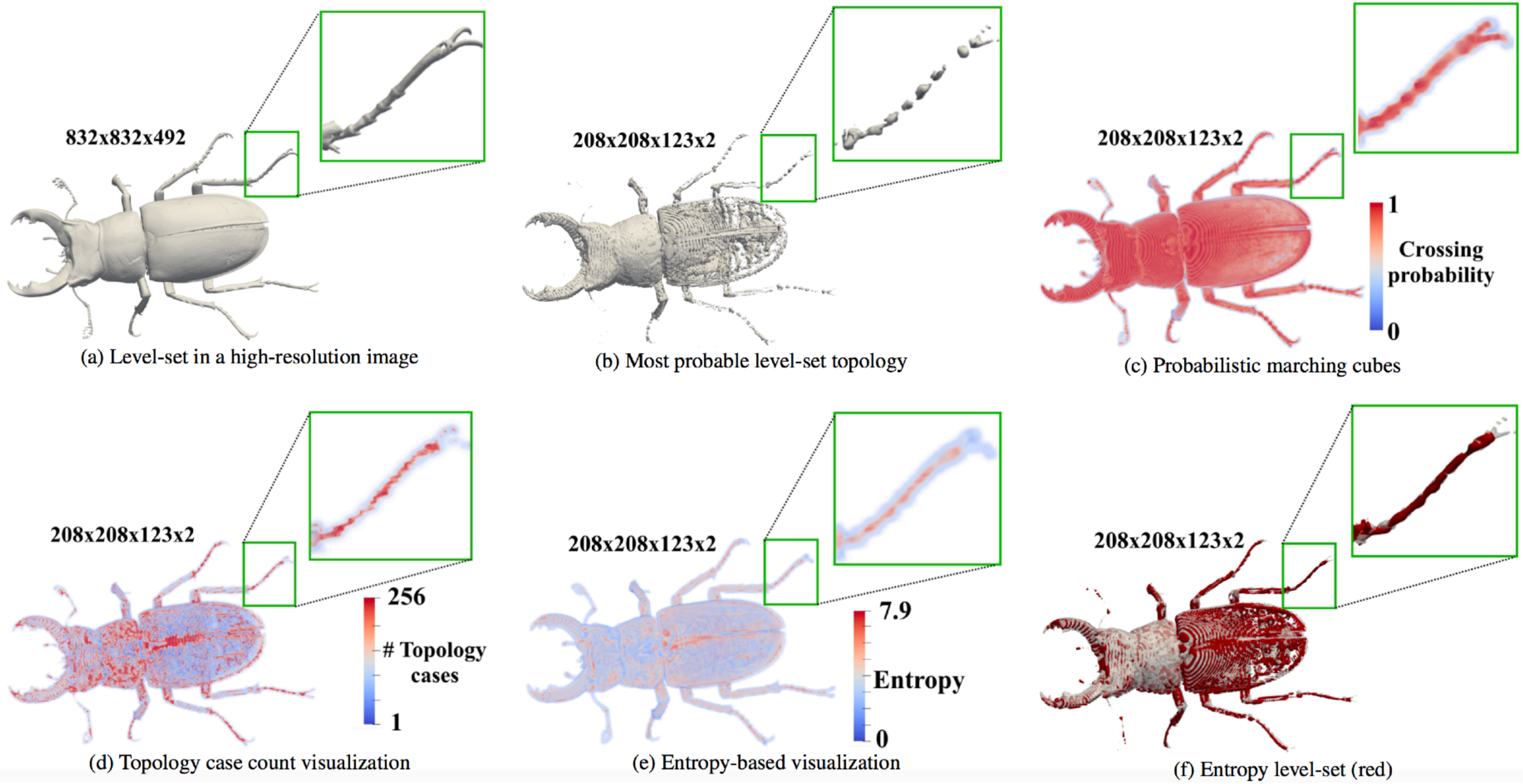


(d) Topology distribution for pixel (39,25) of image (c) (multivariate noise)

Data source [Vitart et al., 2017]: http://iridl.ldeo.columbia.edu/SOURCES/.ECMWF/.S2S/

Beetle Dataset: Marching Cubes Topological Uncertainty

Data source [Gröller et al., 2009]: https://www.cg.tuwien.ac.at/research/vis/datasets/

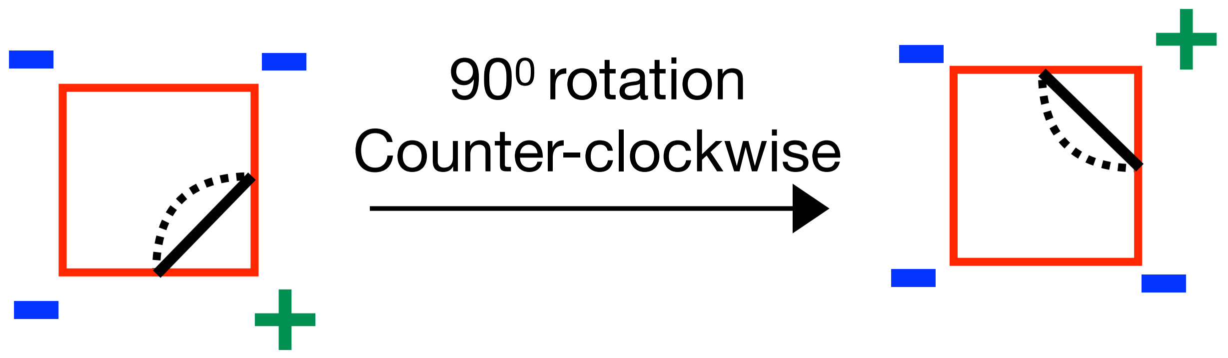


Conclusion

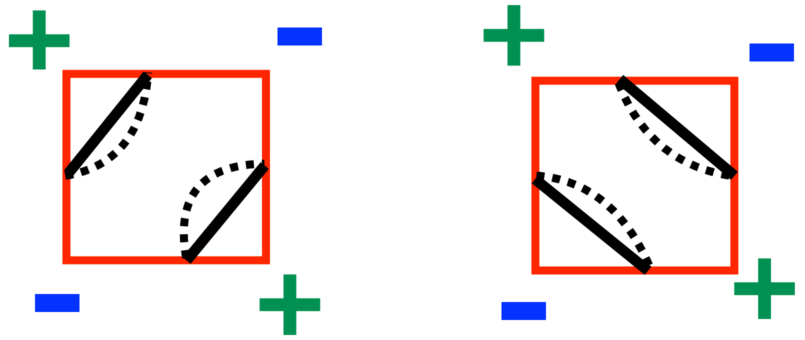
- The study of the uncertainty arising in the marching squares and marching cubes topology cases for uncertain scalar field data
- *Topology case count* and *entropy-based quantification* to capture topological uncertainty of level-sets
- Analysis for the independent and correlated random field assumptions
- Uncertainty visualization with color mapping, interactive probability queries, and entropy isosurfaces

Future Work

- Consider correlation among topology cases for uncertainty quantification



- Consider topological subcases for same sign configuration



- Consider data correlation with neighboring cells for uncertainty quantification

Thank you!

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