VIS2020

Direct Volume Rendering with Nonparametric Models of Uncertainty

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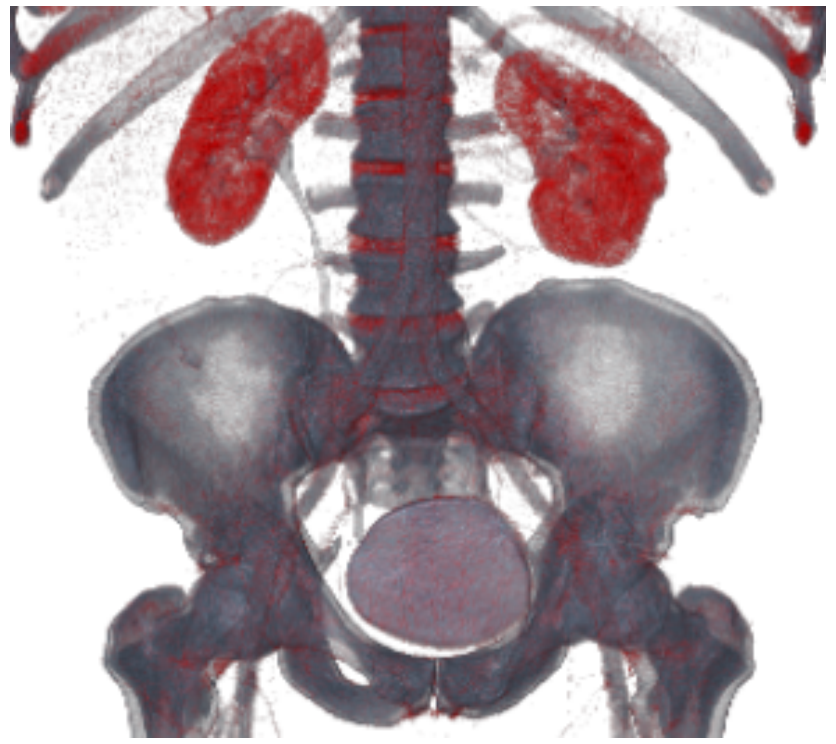
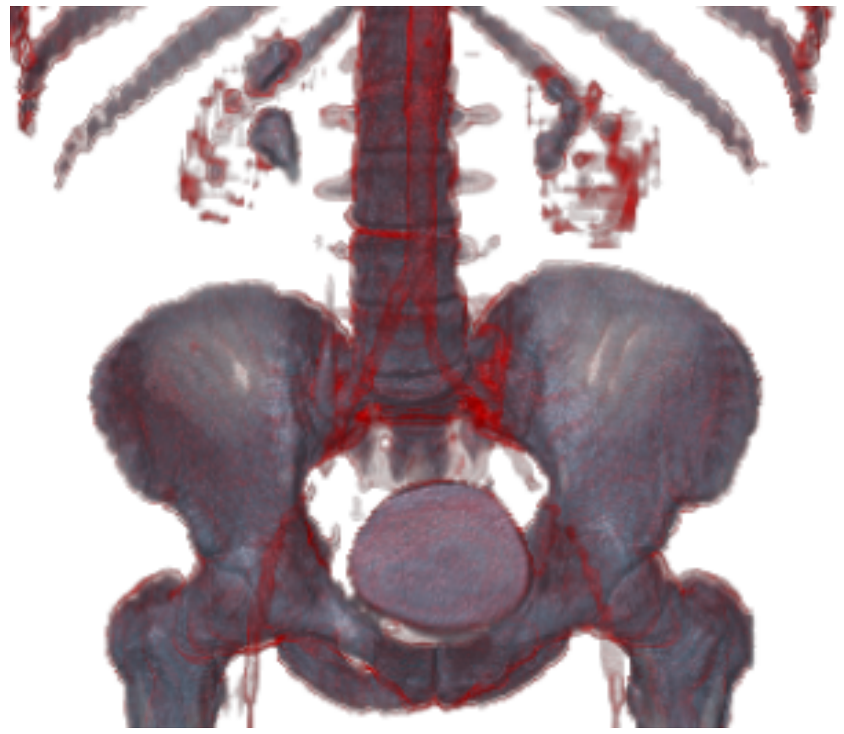
Outline

- "Uncertainty-aware" volume visualizations
  - Related work
  - Parametric models
- Our contributions
  - Nonparametric models
  - 2D transfer functions
- Results, conclusion, and future work



Ground truth



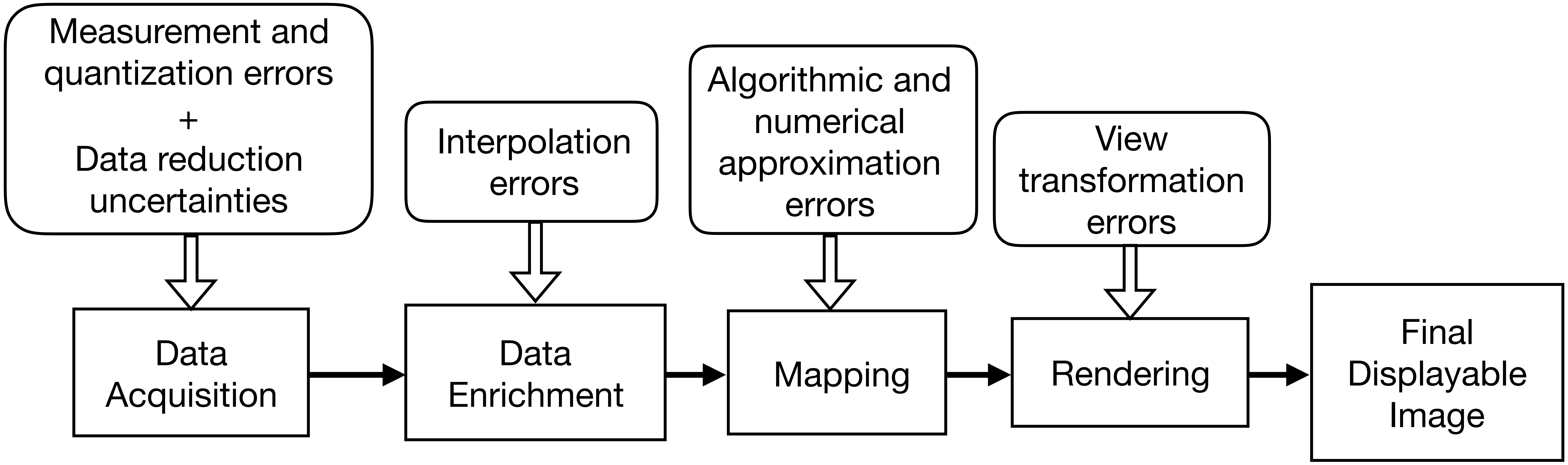
Mean

Parametric
[Sakhaee and Entezari, 2017]

Nonparametric
(proposed)

“Uncertainty-aware” Volume Visualizations

Uncertainty Visualization [Johnson and Sanderson, 2004]

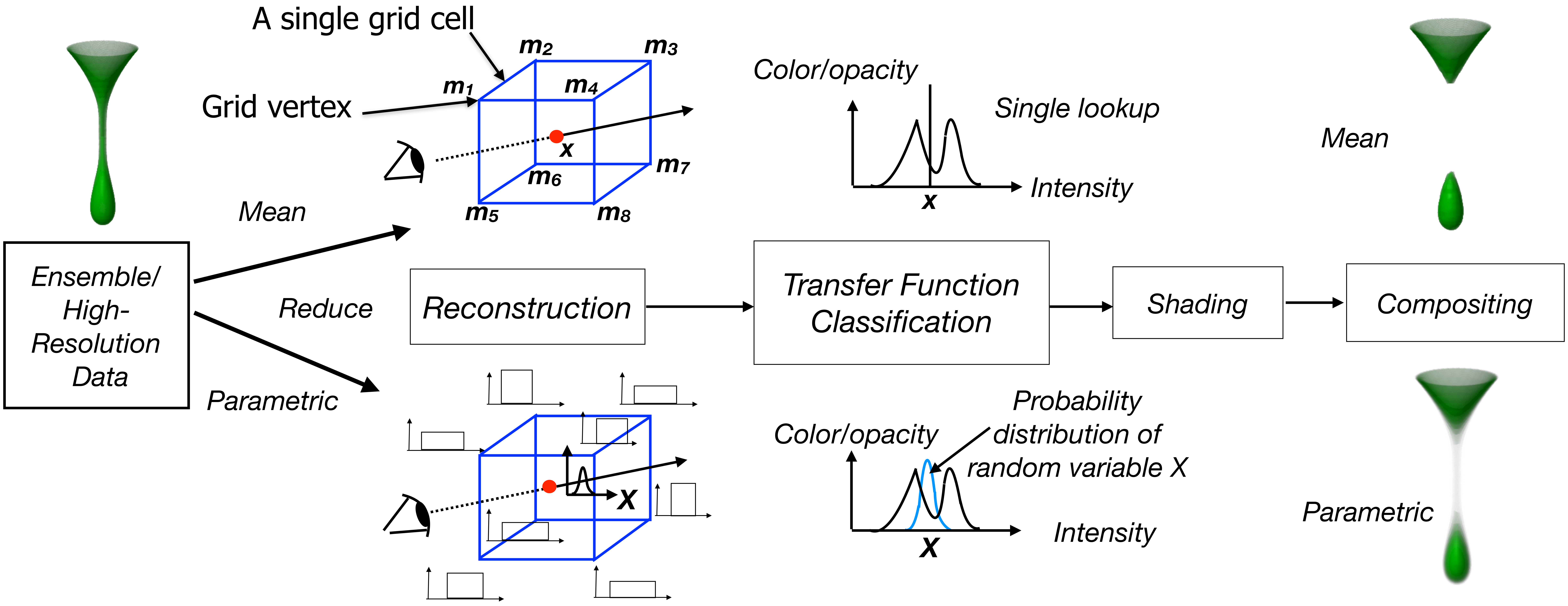


Visualization pipeline
[Brodlie et al., 2012]

Volume Rendering Uncertainty

- Volume visualization via mapping data and uncertainty into color and opacity dimensions of a transfer function, respectively [Djurcilov et al., 2002]
- Statistical quantifications vs. transfer functions for volume visualization [Kniss et al., 2005]
- Study of possible volume renderings via varying uncertain parameters of each stage of volume rendering pipeline [Fout and Ma, 2012]
- Effect of discretization errors on volume rendered images [Etiene et al., 2014, 2015]

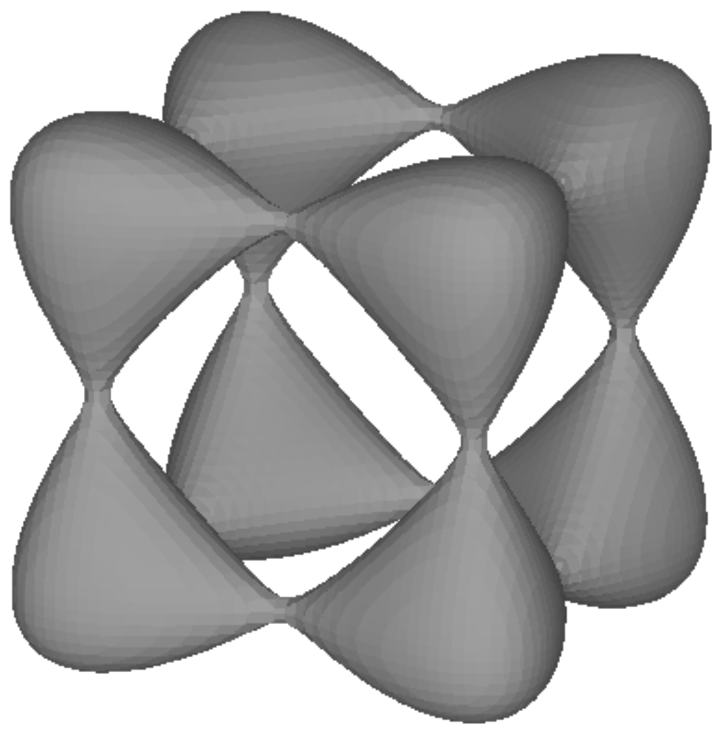
Statistical Volume Rendering: Parametric Models [Sakhaee and Entezari, 2017]



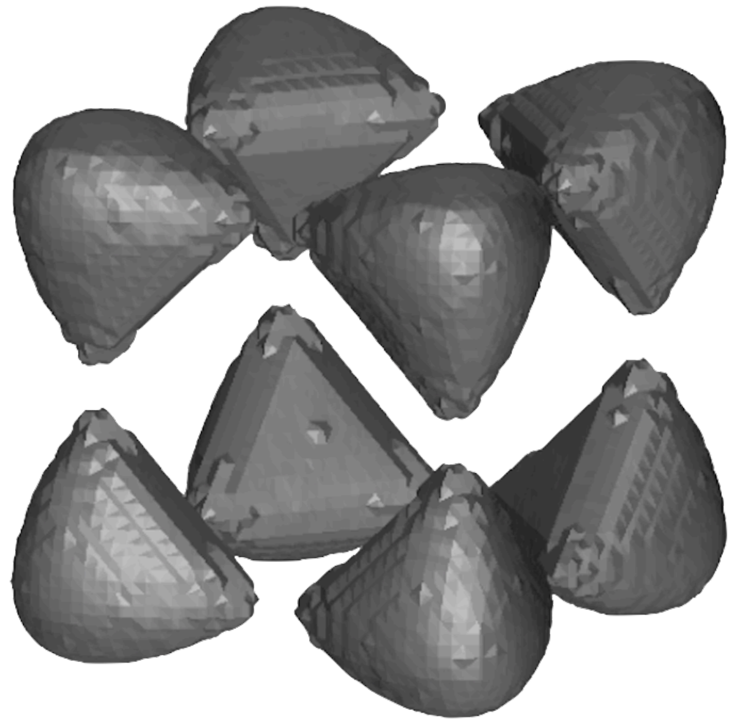
Nonparametric Models of Uncertainty

Nonparametric Models of Uncertainty

Ground truth

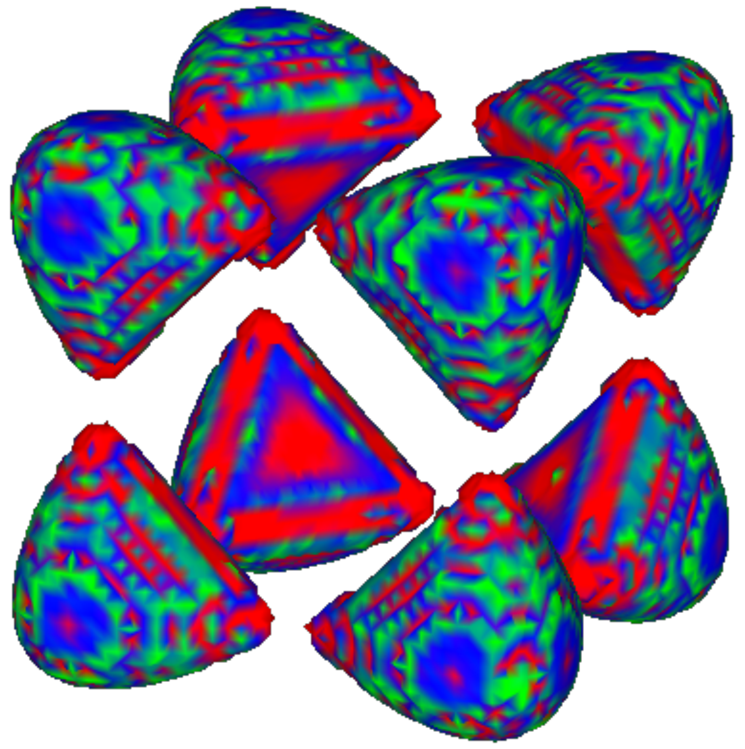


Mean

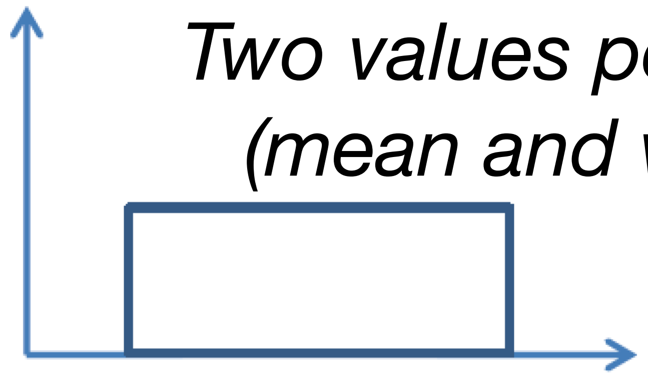


One value per-voxel

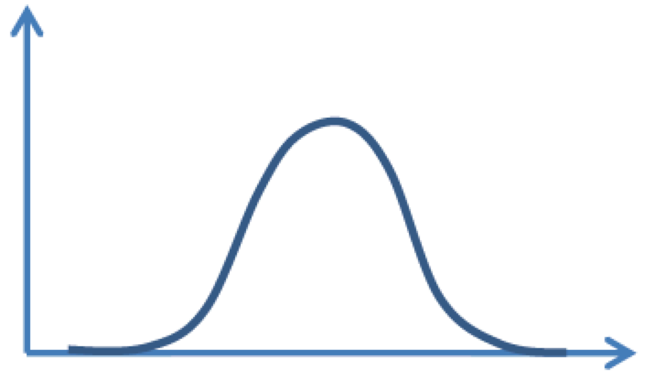
Parametric



Two values per-voxel (mean and width)

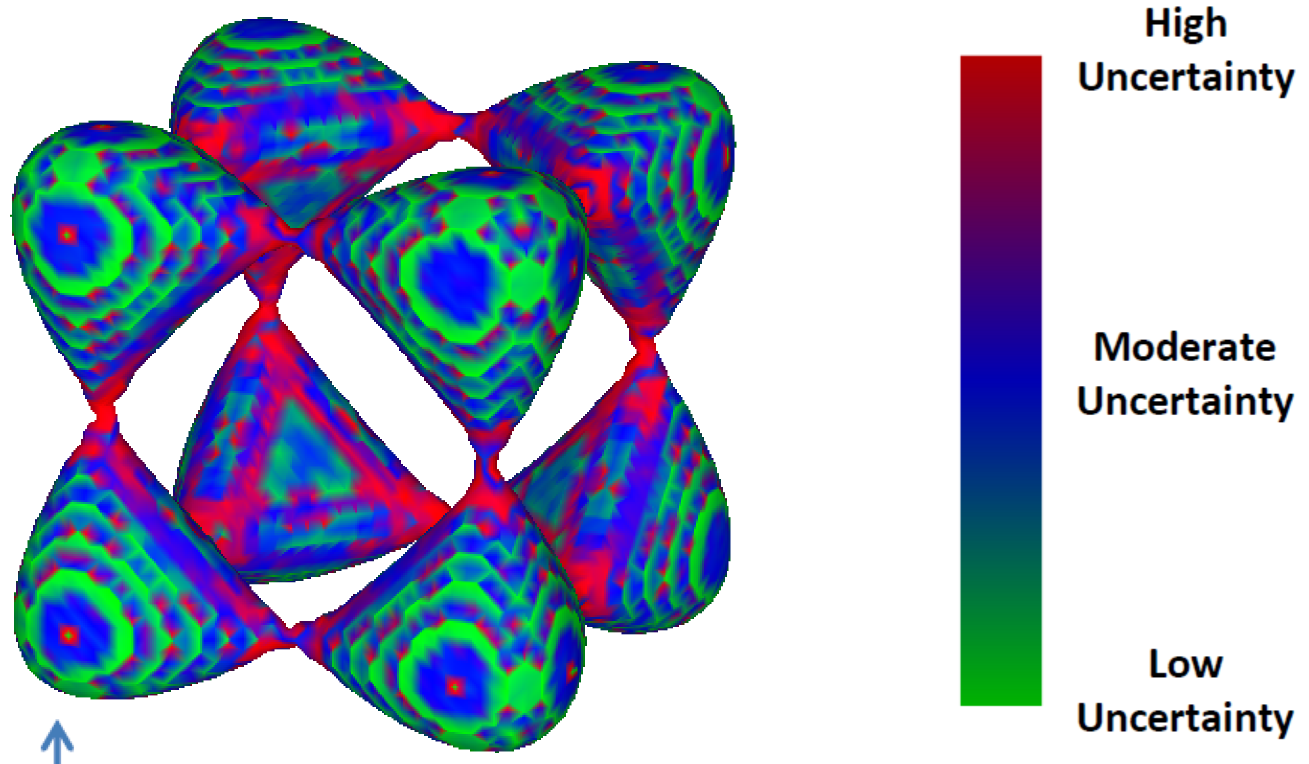


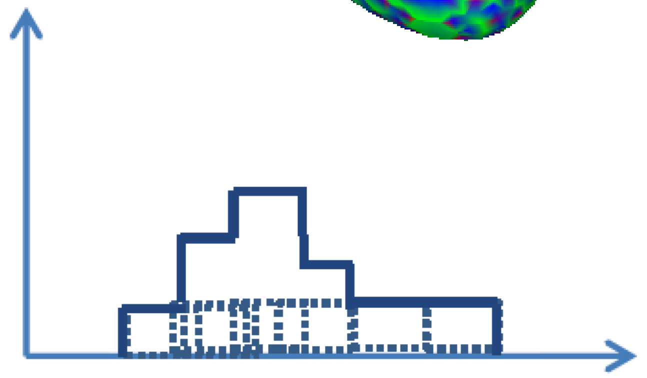
Uniform



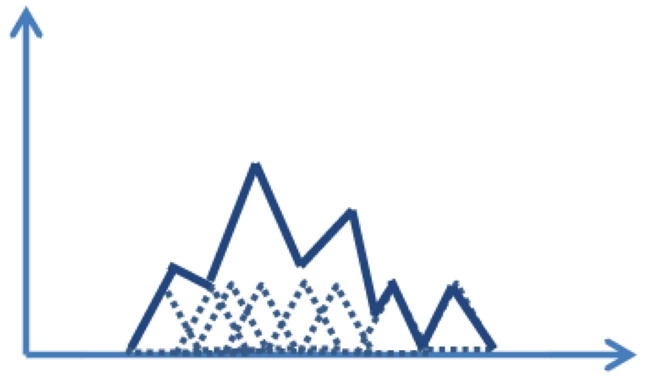
Gaussian

Nonparametric





Uniform kernel



Triangular kernel

n representative samples per-voxel (e.g., min, Q1, median, Q2, max)

Probabilistic marching cubes, [Pöthkow et al., 2013];

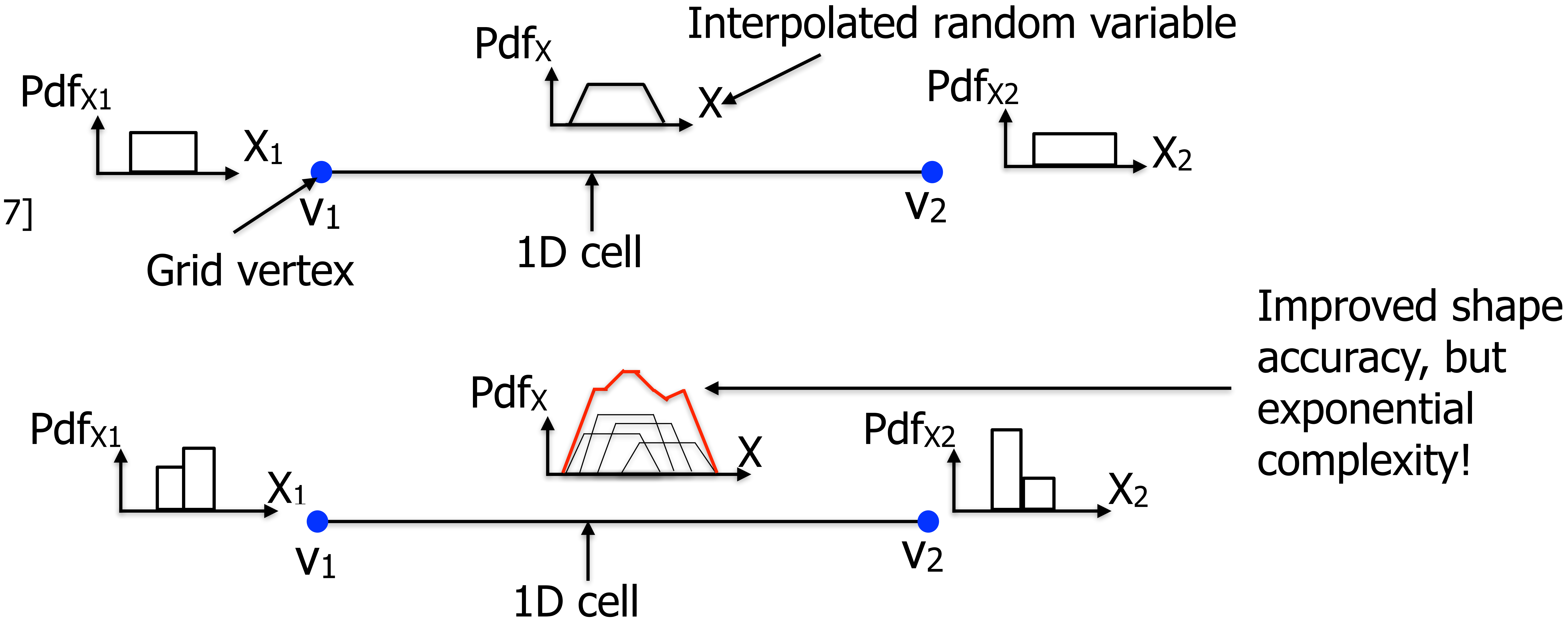
Uncertainty-aware marching cubes [Athawale et al., 2015]

Linear Interpolation of Random Variables

Linear interpolation of random variables represents the convolution of their probability distributions (for independent noise assumption) [Hogg et al., 2004]

Parametric
[Sakhaee and Entezari, 2017]

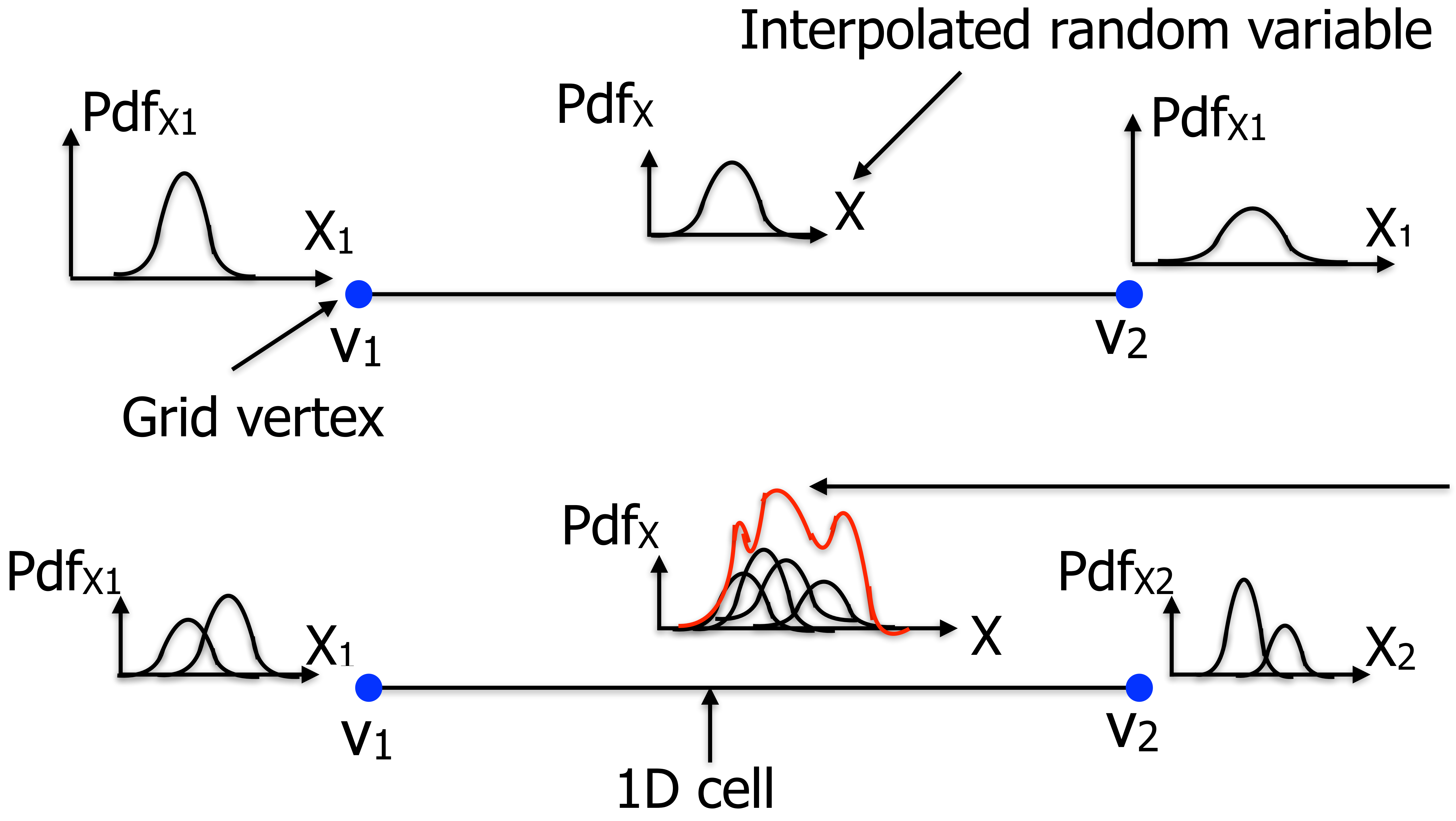
Nonparametric



Linear Interpolation of Random Variables

Gaussian
[Sakhaee and Entezari, 2017; Feller, 1968]

Gaussian mixtures



Improved shape accuracy, but exponential complexity!

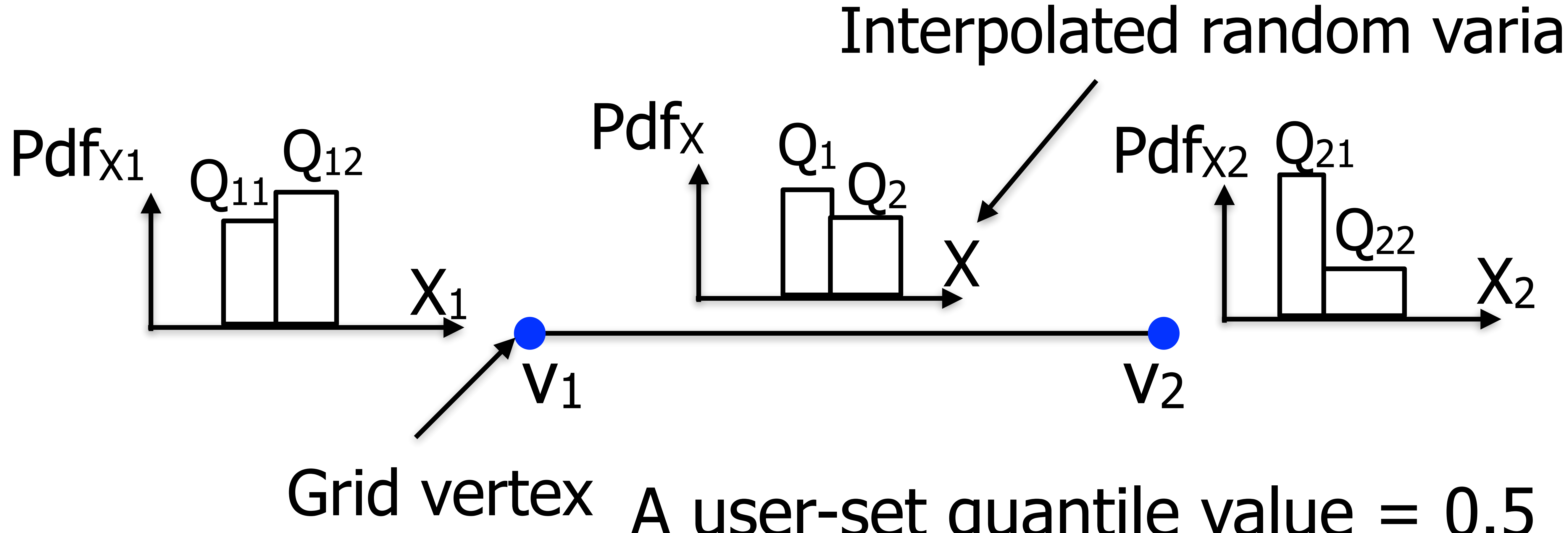
- Monte Carlo sampling of Gaussian mixtures [Liu et al., 2012]
- Image space compositing of Monte Carlo visualizations

Quantile Interpolation [Read, 1999] for Nonparametric Distributions

- Closed-form linear time complexity framework (No Monte Carlo sampling, non-exponential framework)
- Object space compositing (No image space compositing)

Q_{ij}: Quantile j for a random variable X_i

Q_j: Quantile j for interpolated random variable X



A user-set quantile value = 0.5

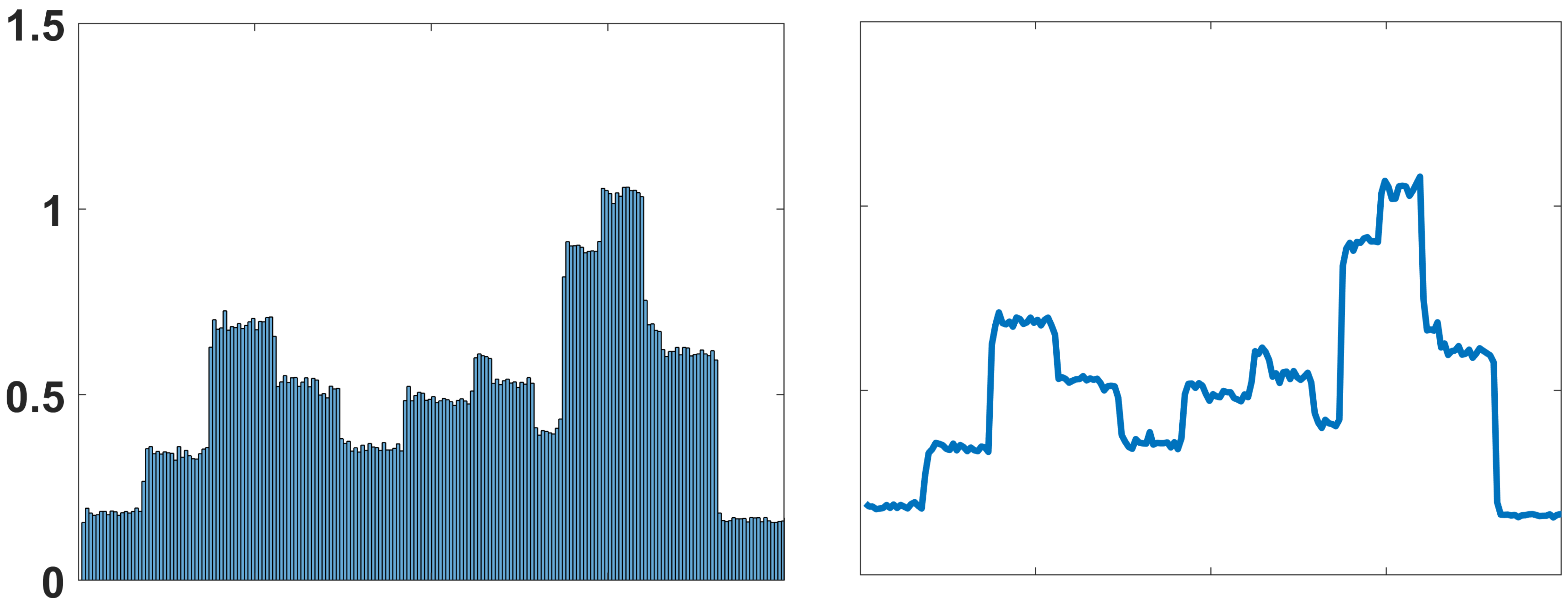
Interpolation of the j'th quantile:

$$Pr(Q_j) = \frac{Pr(Q_{1j})Pr(Q_{2j})}{(1-\alpha)Pr(Q_{2j}) + \alpha Pr(Q_{1j})}$$

Complexity linearly proportional to the number of quantiles

Quantile Interpolation

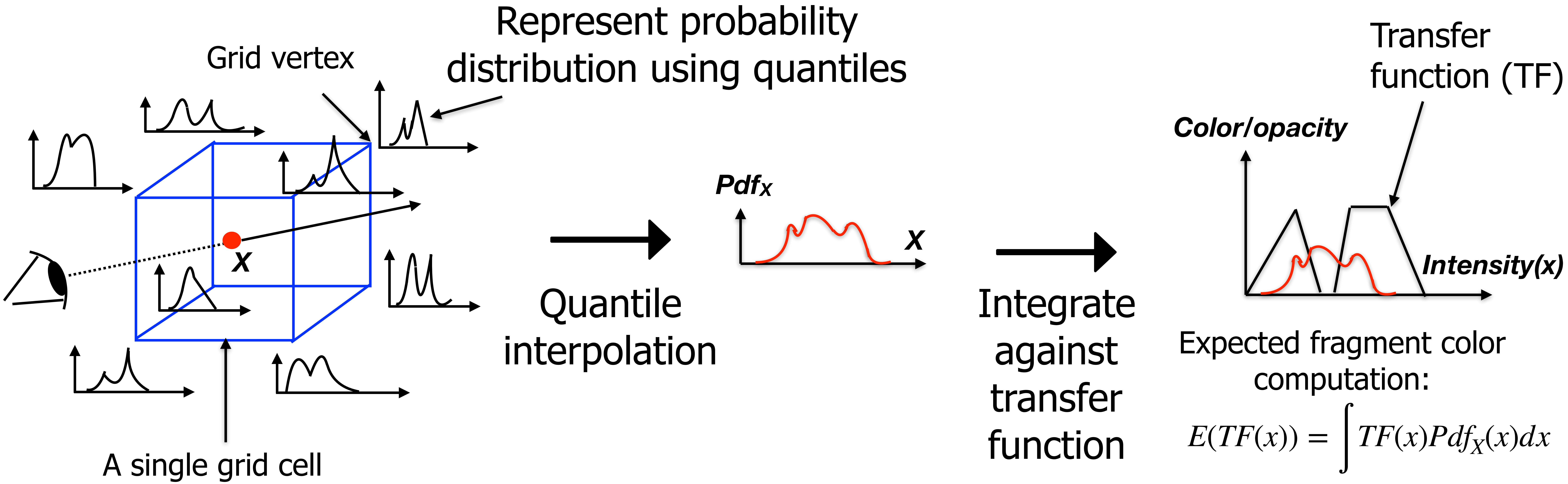
- Reconstruction of 2D uncertain vector fields [Hollister and Pang, 2013, 2015]
- We propose reconstruction of 3D uncertain intensity field (refer to the paper for the proof)



Monte Carlo　　Analytical

Probability distribution of interpolated random variable X using quantile interpolation in 3D

Our pipeline for Volume Rendering With Nonparametric Statistics

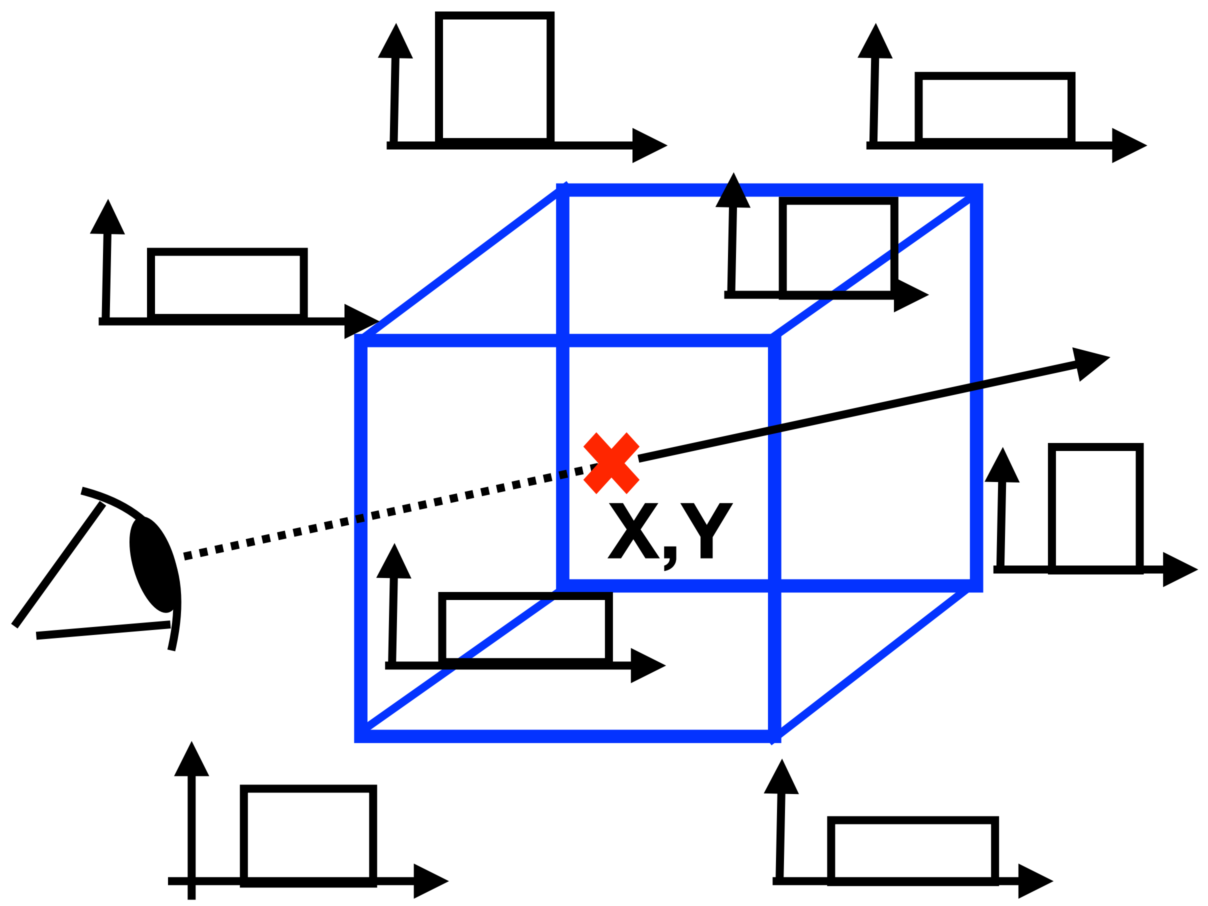


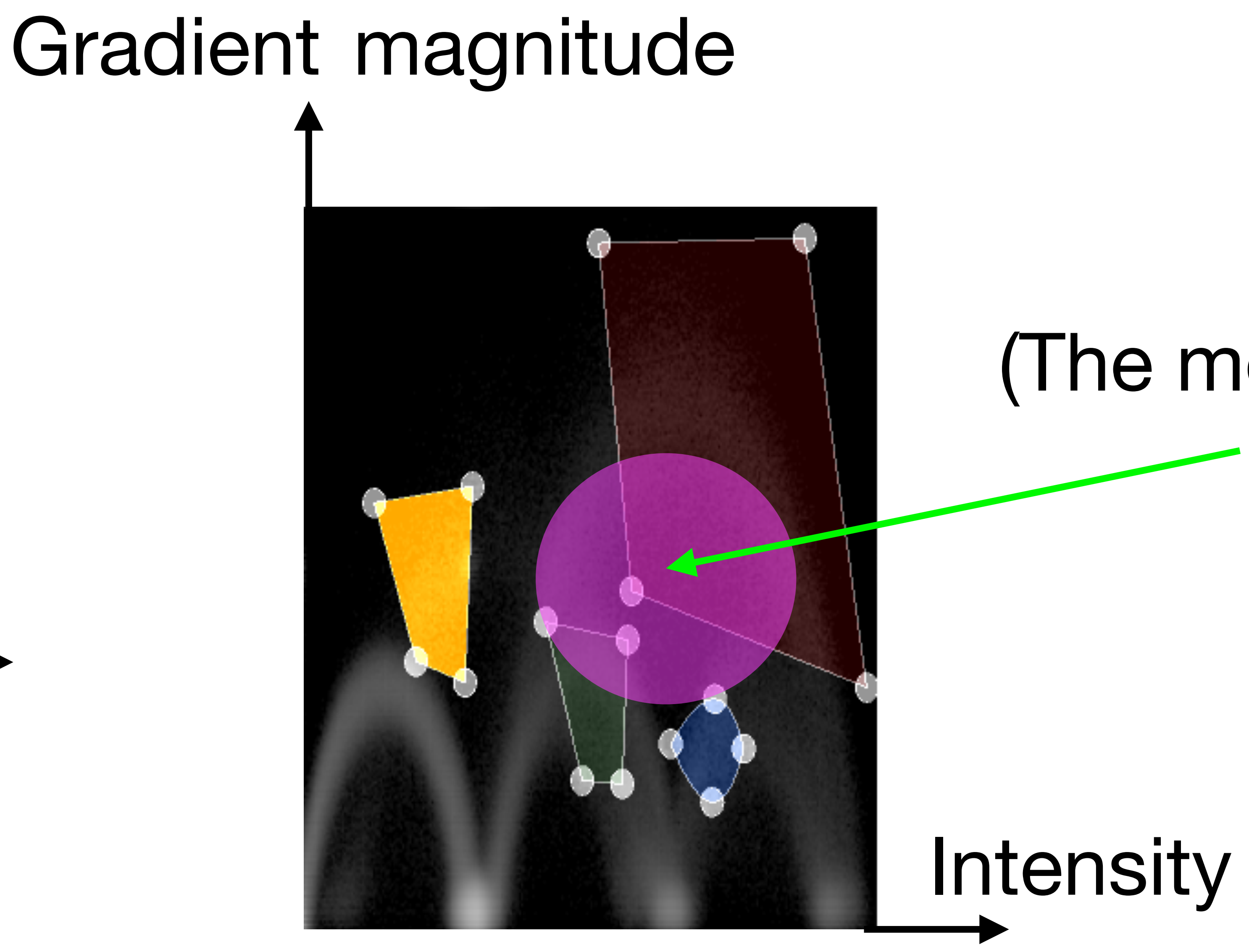
Expected fragment color computation:

$$E(TF(x)) = \int TF(x)Pdf_X(x)dx$$

Statistical Volume Rendering: 2D Transfer Functions

Uncertain field [Sakhaee and Entezari, 2017] + 2D TFs [Kniss et al., 2001]



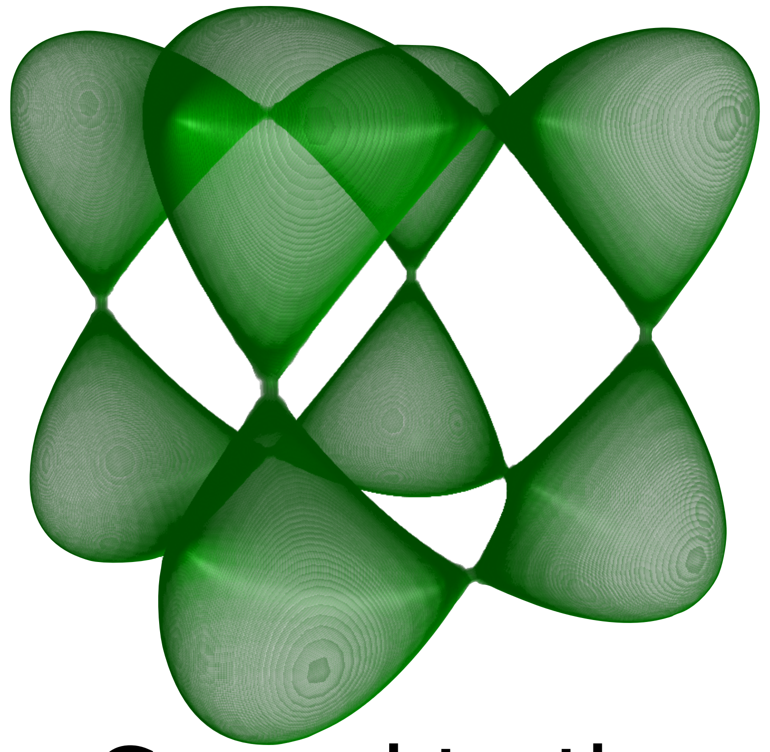


Joint Probability density of X,Y
(The most likely material for sample (X,Y) is red)

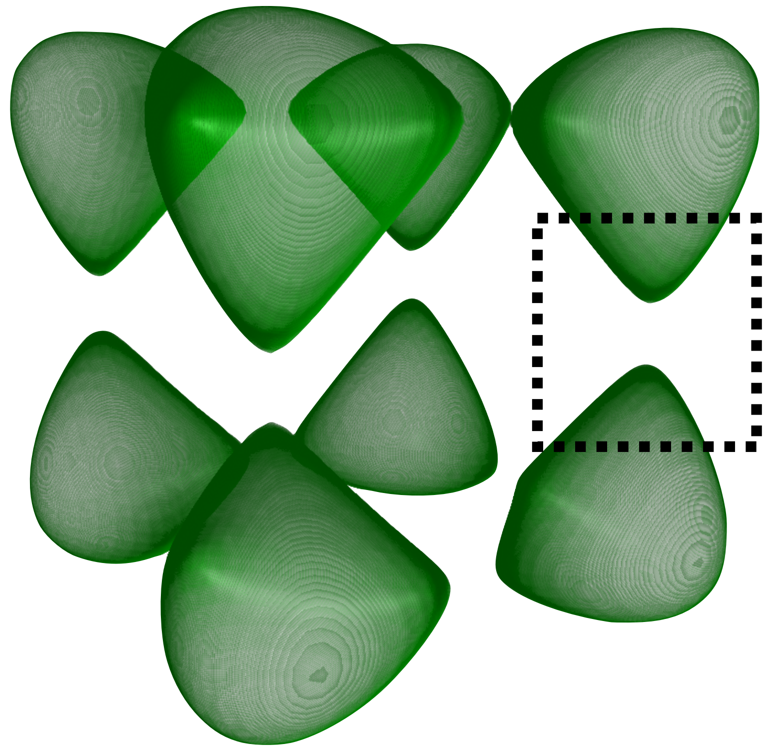
$$E(TF(x,y)) = \int\int TF(x,y)Pdf_{X,Y}(x,y)dxdy$$

Results, conclusion, and future work

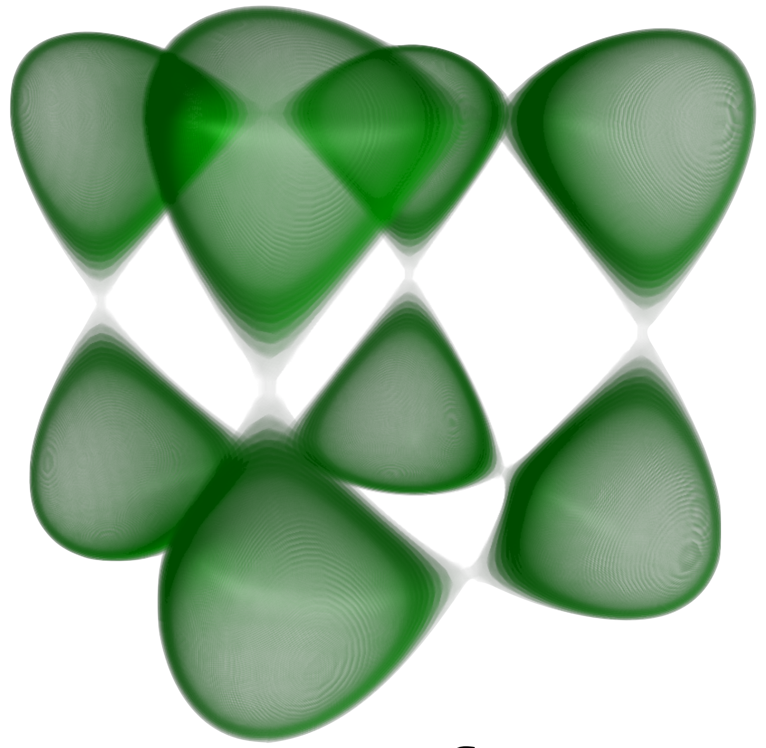
Tangle Function [Knoll et al., 2009]
(Qualitative Comparisons)



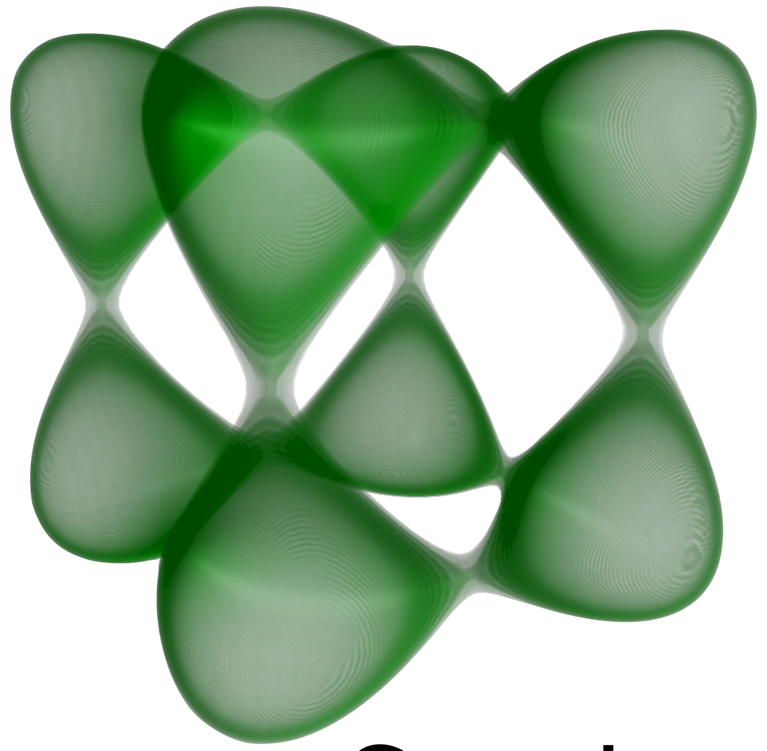
Ground truth



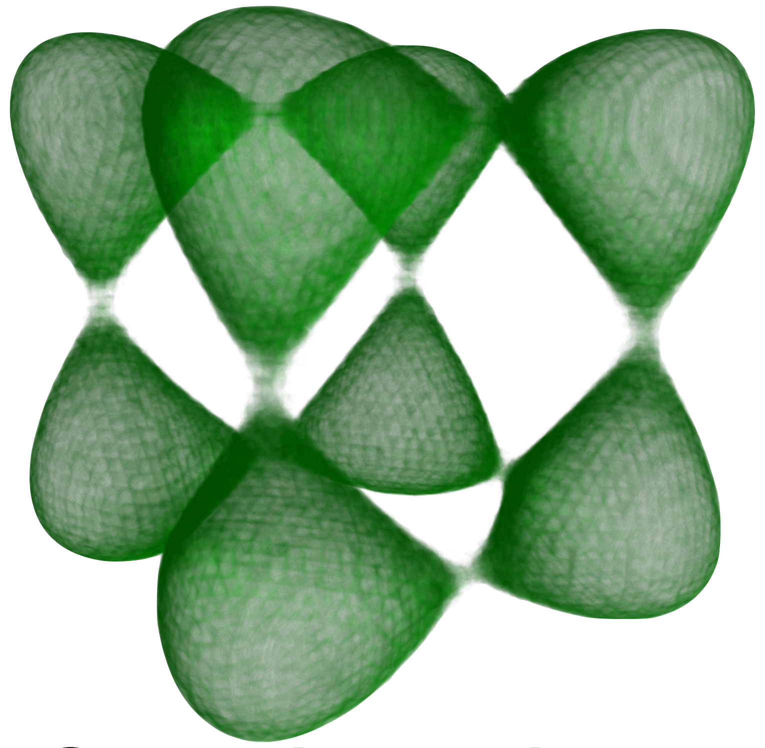
Mean



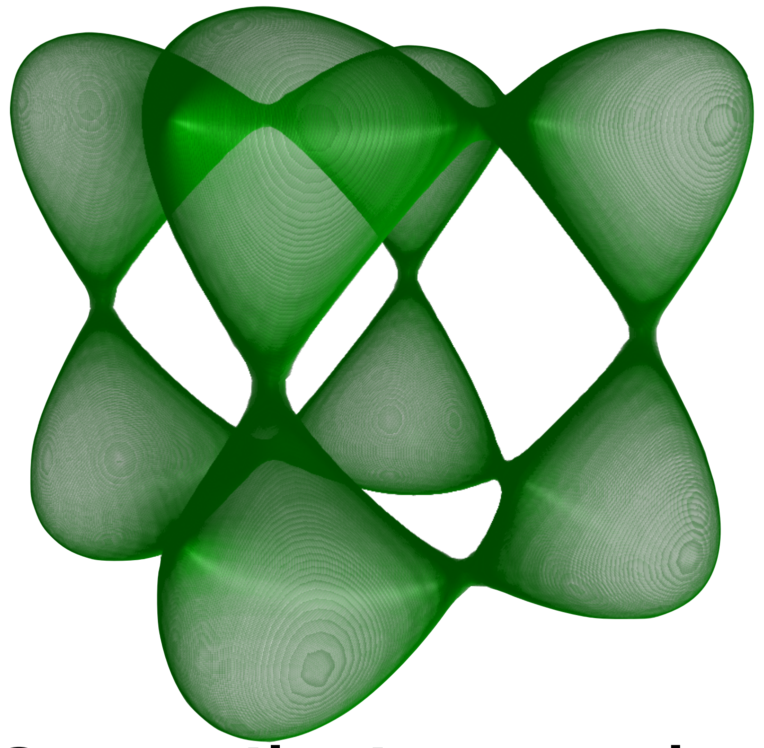
Uniform



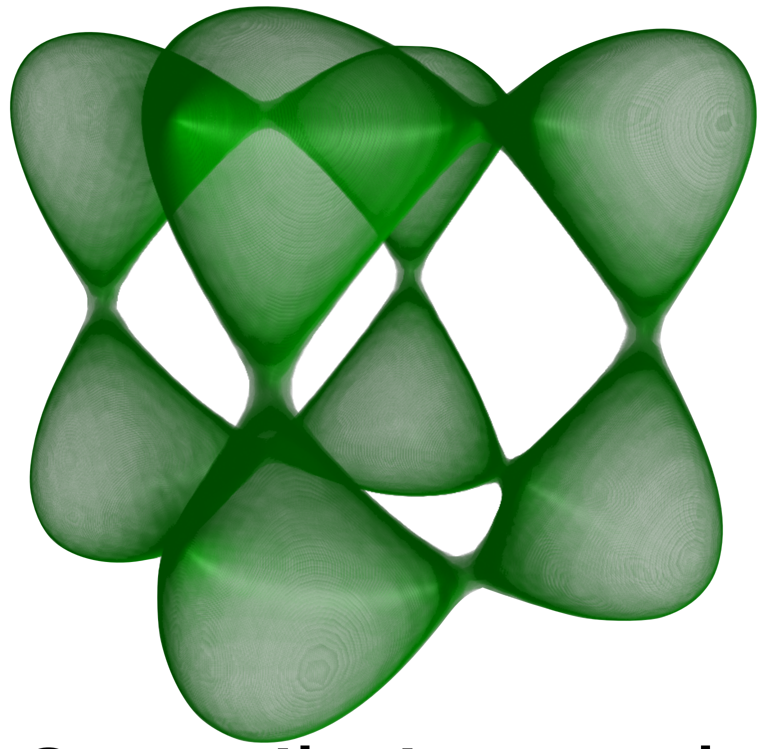
Gaussian



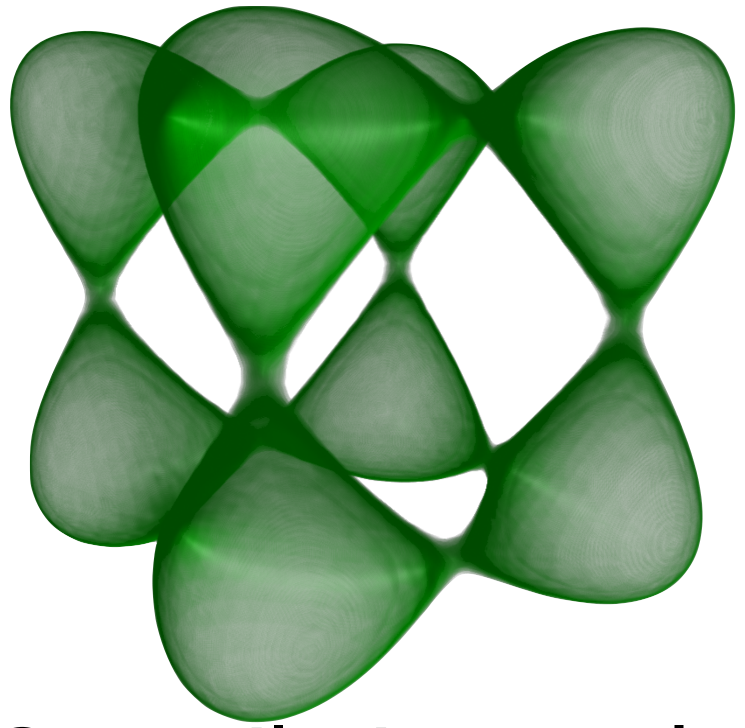
Gaussian mixtures
(four Gaussians)
(Monte Carlo)



Quantile interpolation
(two quantiles)

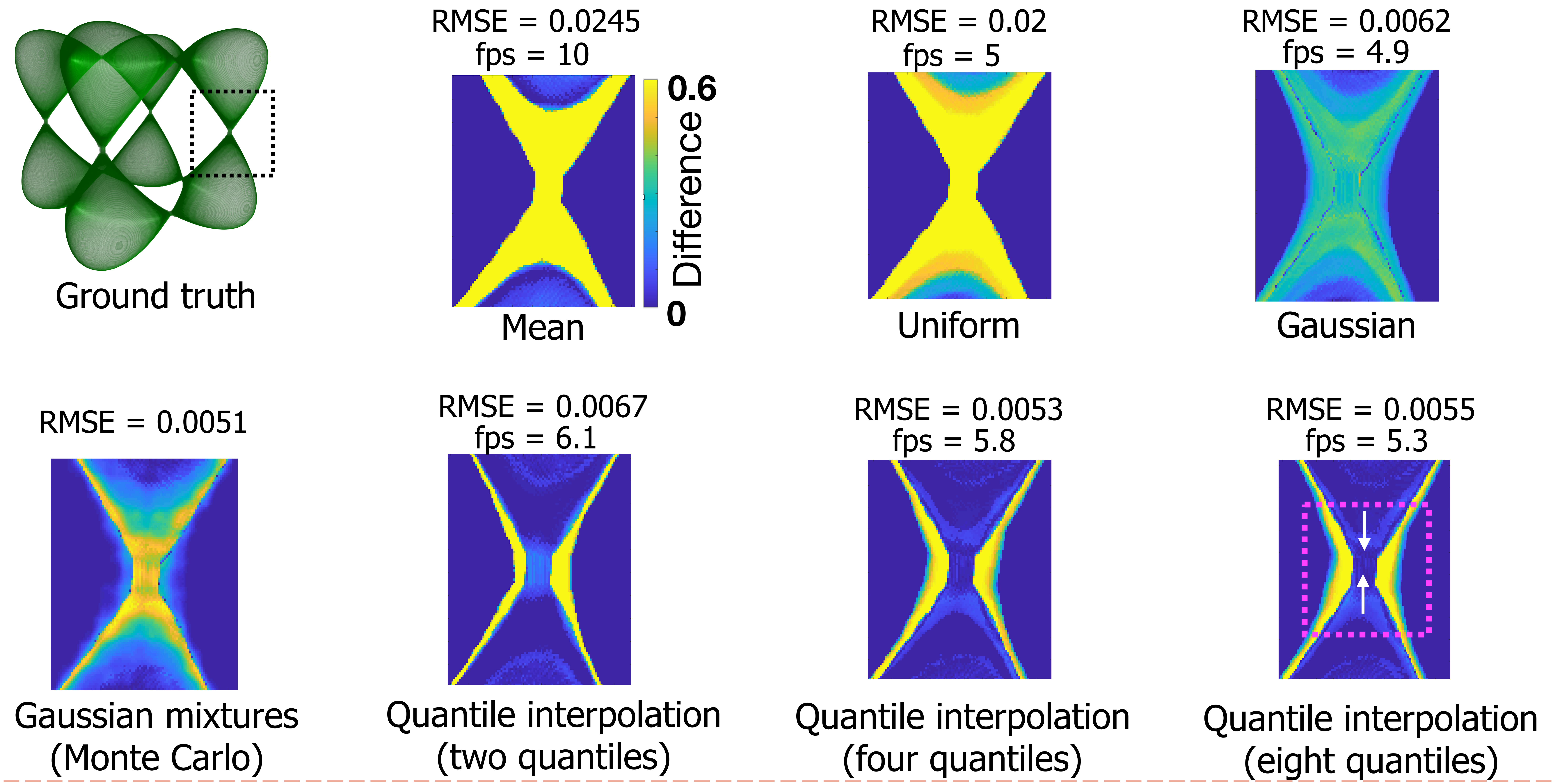


Quantile interpolation
(four quantiles)

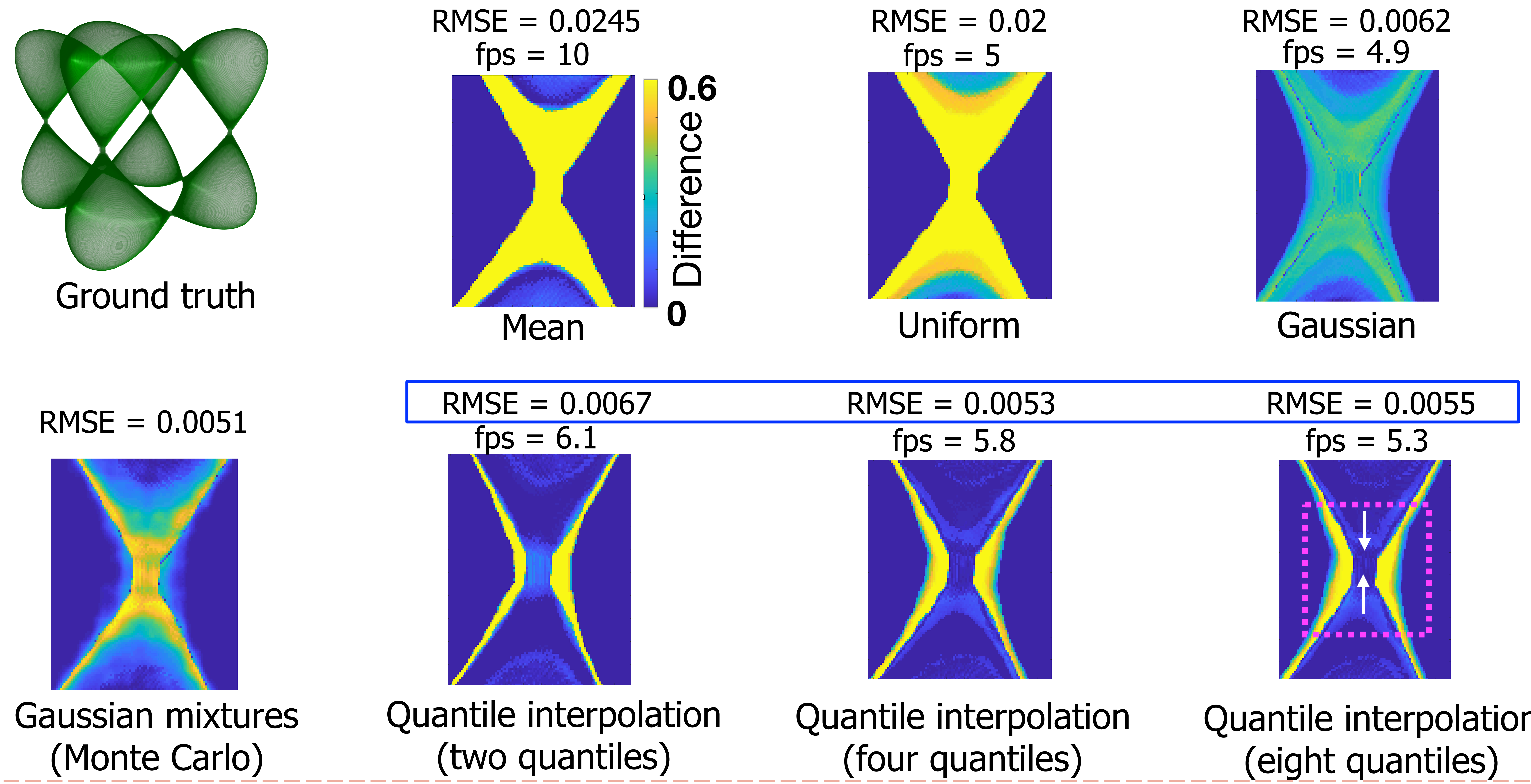


Quantile interpolation
(eight quantiles)

Tangle function (Quantitative Comparisons)



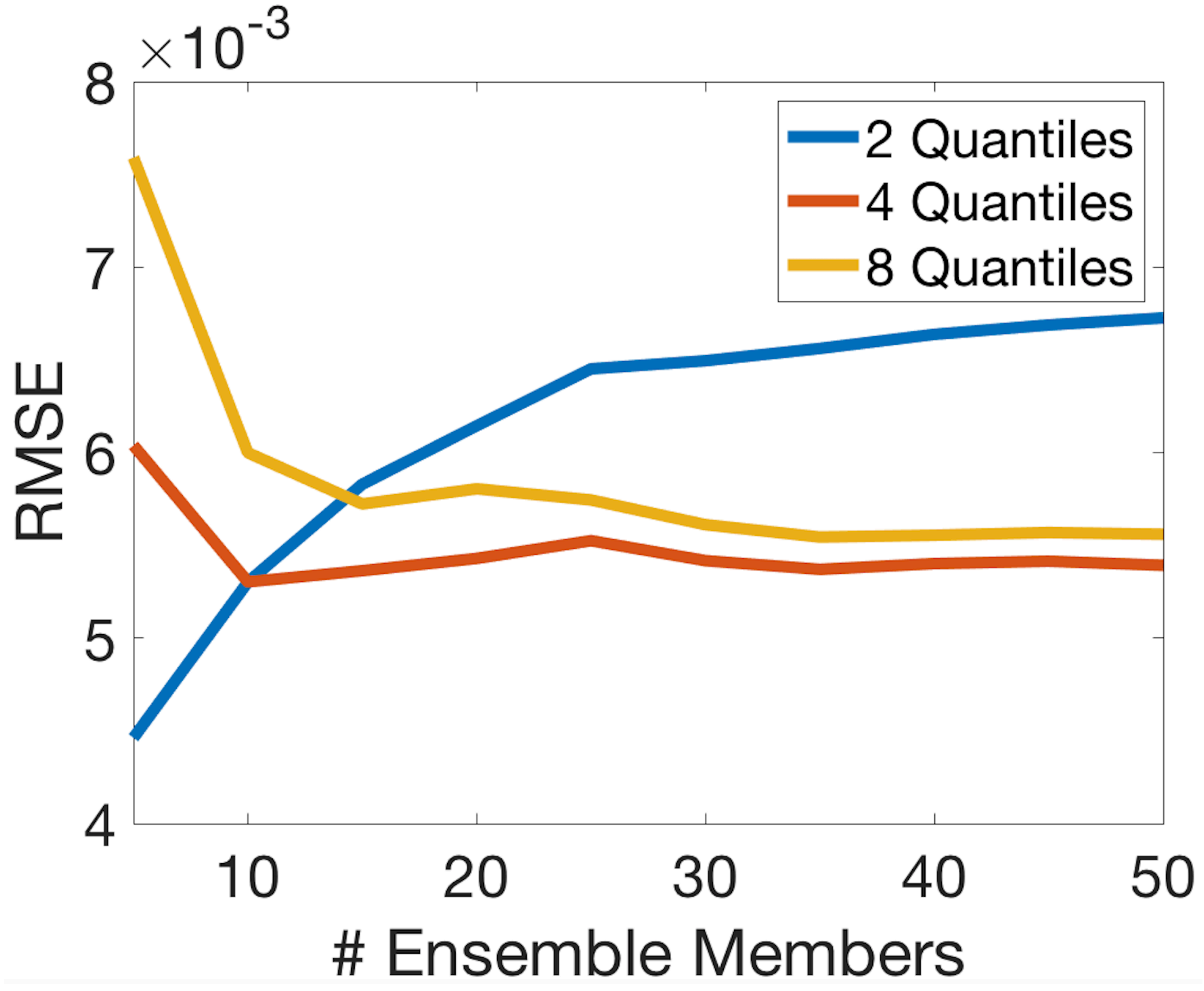
Tangle function (Quantitative Comparisons)



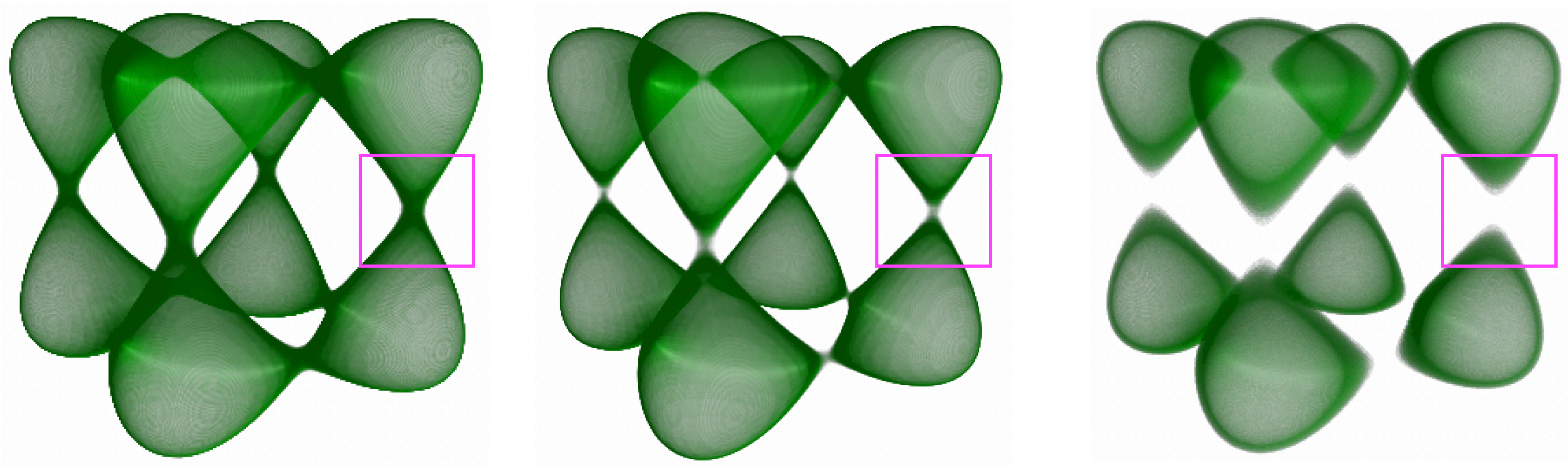
Number of Quantiles for Quantile Interpolation

Depends on sample size (#ensemble members), available memory, and computational power.

Tangle function



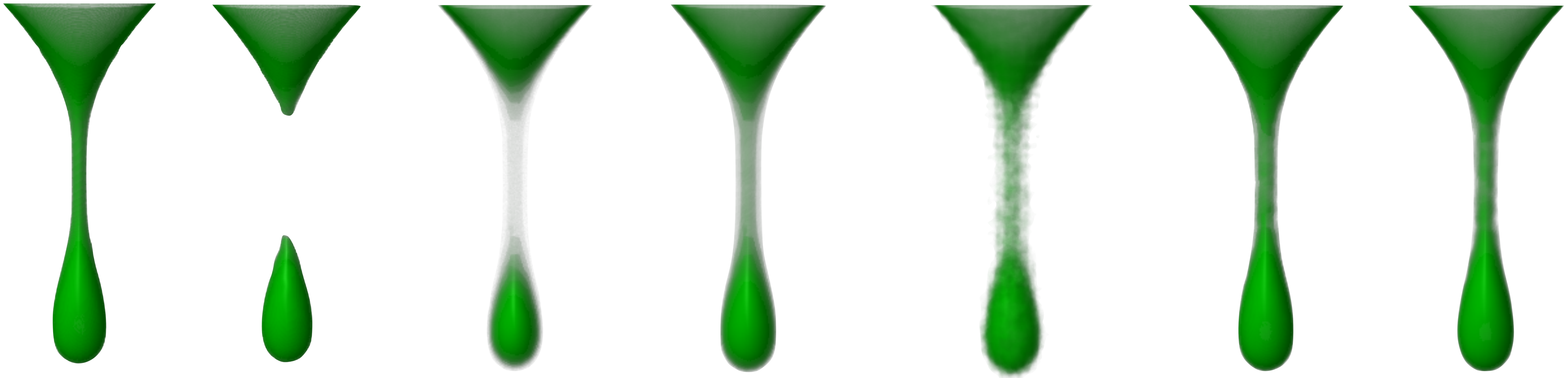
Quartile View: Uncertainty Visualization



(a) Lower quartile (b) Central 50%, IQR (c) Upper quartile

Teardrop Function [Knoll et al., 2009] (Qualitative Comparisons)

[Sakhaee and Entezari, 2017] [Liu et al., 2012] [Proposed]



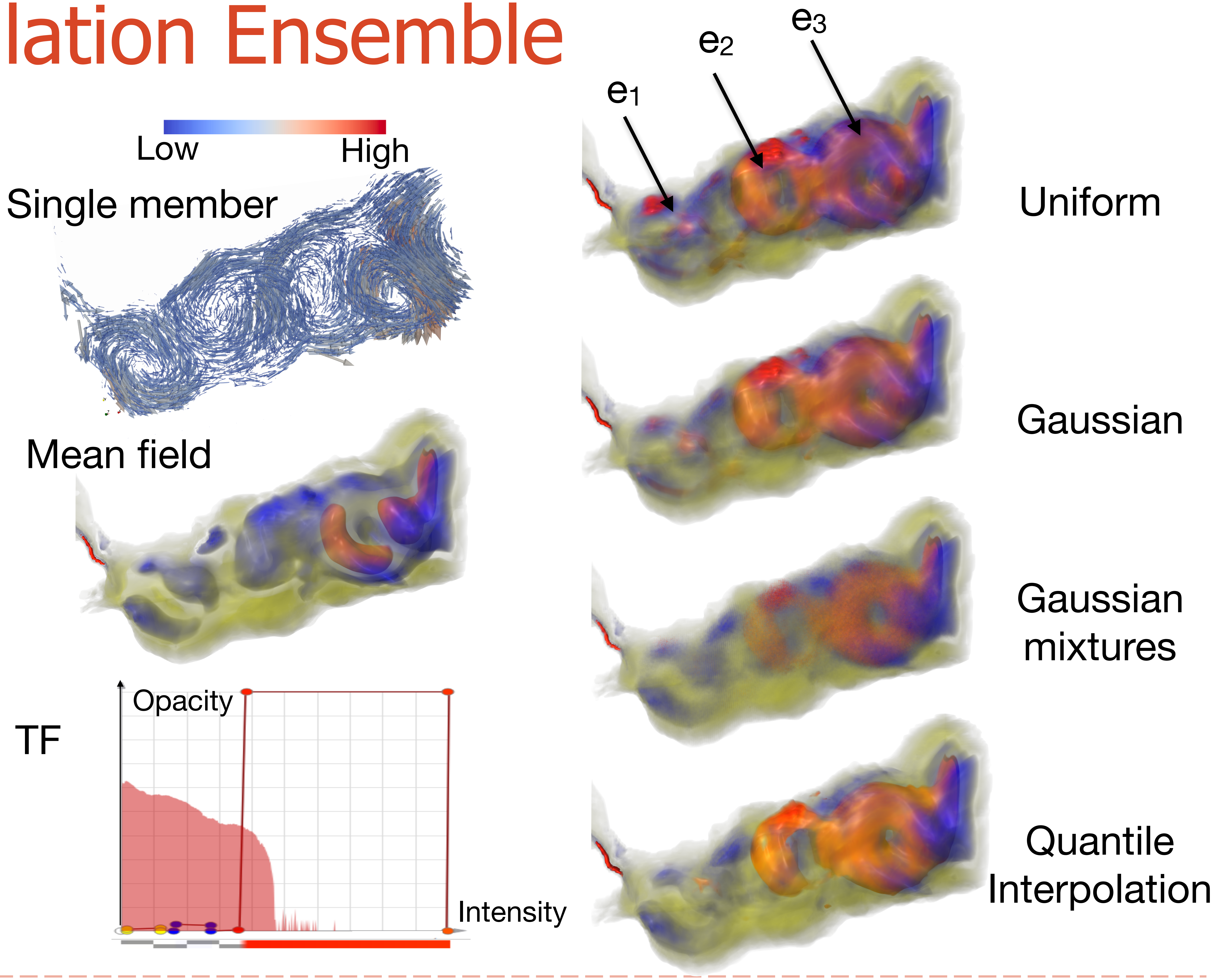
Ground truth | Mean | Uniform | Gaussian | Gaussian mixtures (Monte Carlo) | Quantile interpolation (four quantiles) | Quantile interpolation (eight quantiles)

Teardrop Function [Knoll et al., 2009] (Quantitative Comparisons)

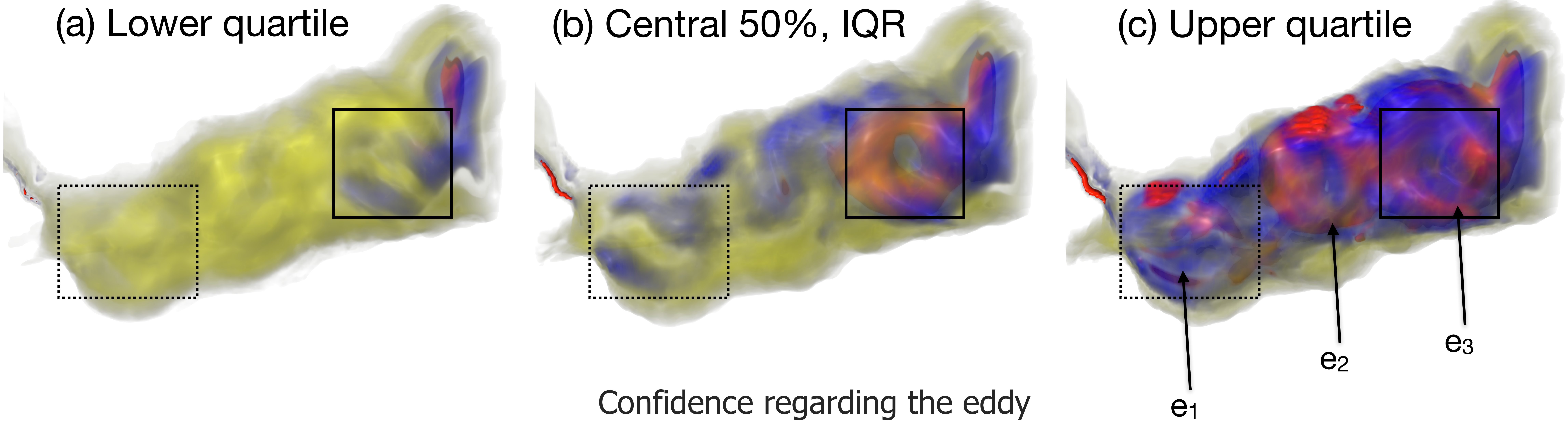
Red Sea Eddy Simulation Ensemble

Courtesy: SciVis Contest 2020 dataset
(https://kaust-vislab.github.io/SciVis2020/)

- Visualization of uncertain velocity magnitude field
- Confidence regarding the eddy presence/position:
  e_3: High
  e_2: Moderate
  e_1: Low



Quartile View: Uncertainty Visualization



Confidence regarding the eddy presence/position:

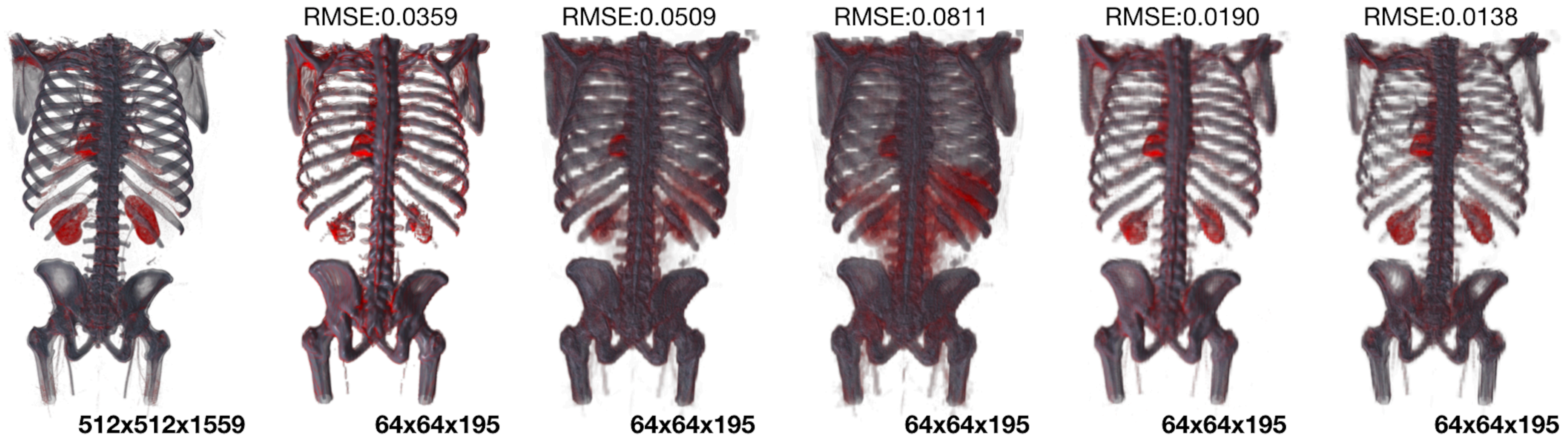
e_3: High

e_2: Moderate

e_1: Low

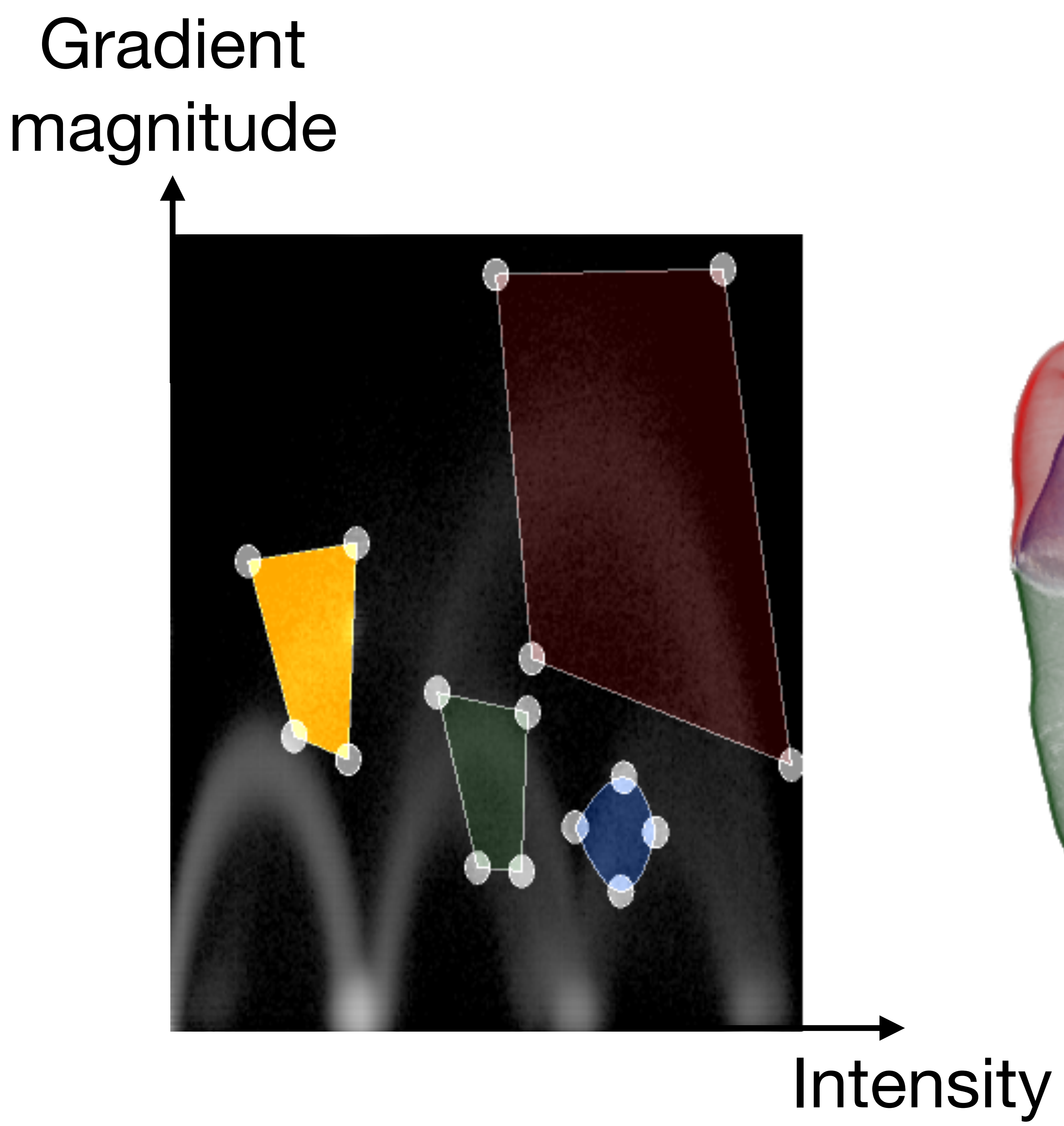
Osirix OBELIX Dataset

Analysis of the uncertainty due to downsampling of data



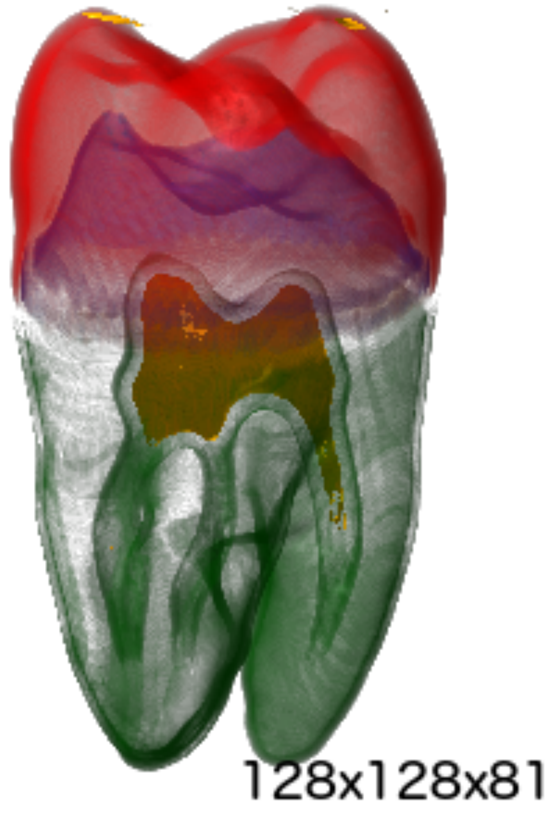
Ground truth | Mean | Uniform | Gaussian | Gaussian mixtures (Monte Carlo) | Quantile interpolation

Statistical Volume Rendering: 2D Transfer Functions

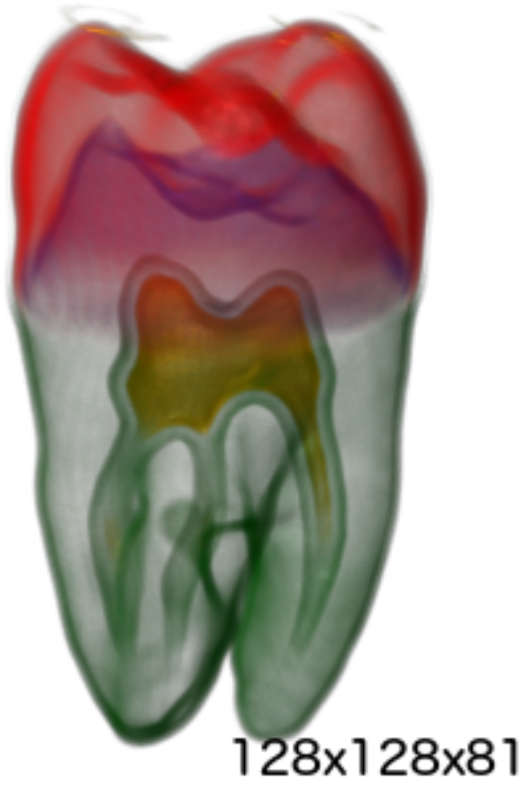


The tooth dataset

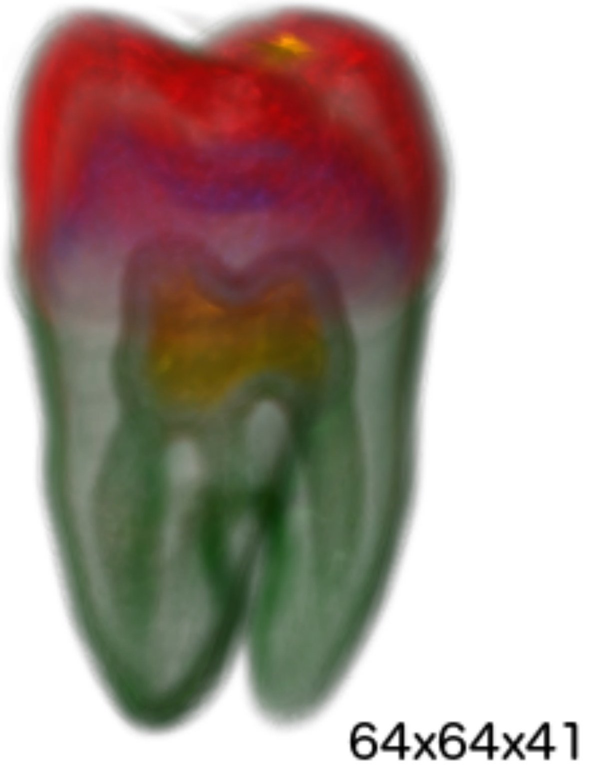
Reduce



Mean



Parametric



Conclusion

- Closed-form nonparametric framework for efficient statistical rendering
- Quantile interpolation for reconstruction
- Qualitative and quantitative comparisons with the mean, parametric, and Gaussian mixture models
- Application of statistical volume rendering to 2D transfer functions

Future Work

- Uneven quantile values for quantile interpolation
- Multidimensional transfer functions (more than two dimensions) for nonparametric models
- Dependent random fields
- Optimal parameter estimation for noise models

Thank you for your attention!

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