Uncertainty Visualization of 2D Morse Complex Ensembles Using Statistical Summary Maps

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Supplementary Material

Kármán Vortex Street Dataset

We analyze Morse complexes from an ensemble of Kármán vortex street simulations. The original flow simulation of the Kármán vortex street dataset is available via the Gerris software [1]; it is generated as the result of a steady flow (moving from left to right) obstructed by an obstacle situated at the far left. We generate 15 simulations of the Kármán vortex street with Gerris; each simulation is run for a fixed amount of time with varying fluid viscosity parameter. We then obtain an ensemble of scalar fields computed as the magnitude of the flow velocity. We are interested in capturing structural variation of Morse complexes that arise from these scalar fields, which are associated with the vortical structures of the data. The analysis of vortical structures and their uncertainty has important applications in structural engineering [2].

We first compute the mean field for the ensemble and visualize it in Fig. 1a. Red regions indicate vortical structures in the flow.

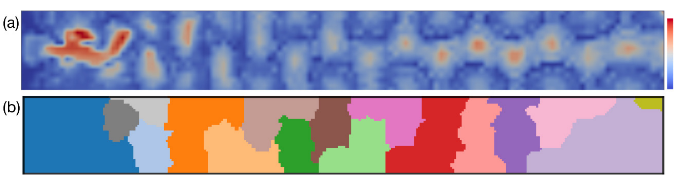

Fig. 1. Kármán vortex street dataset: (a) Mean field. (b) The Morse complex of the mean field after persistence simplification with 17 maxima.

Persistence simplification. Fig. 2a visualizes the persistence graphs for the Kármán vortex street ensemble dataset, which do not contain an obvious plateau like the one we observed for the wind dataset. However, at the selected simplification scale (dotted red line), the ensemble members show an agreement in the number of maxima (17, dotted pink line). Furthermore, the topology of the 1-cells across the simplified Morse complexes appears consistent in the spaghetti plots (Fig. 2b). Therefore, we simplify each ensemble member at the selected scale with 17 maxima. Fig. 3 visualizes the Morse complexes for two ensemble members before and after simplification. The spatial variations of 2-cell boundaries appear to be substantial, even after simplification. In comparison, the Morse complex of the mean field (after persistence simplification at the selected scale) fails to capture such spatial variations (Fig. 1b).

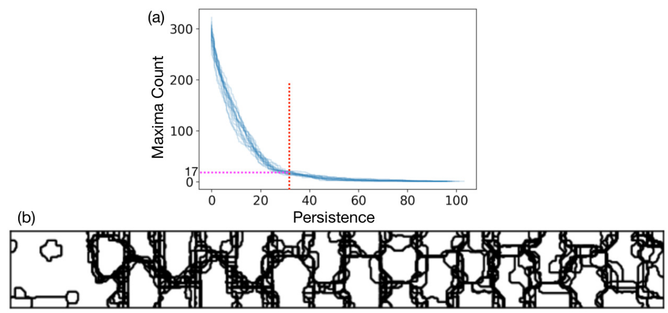

Fig. 2. Kármán vortex street dataset: persistence simplification. (a) Persistence graphs. (b) Spaghetti plots of the simplified Morse complexes.

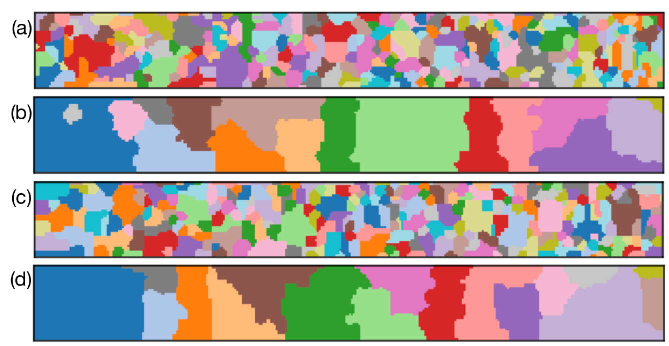

Fig. 3. Kármán vortex street dataset: Morse complexes of two ensemble members before (a, c) and after (b, d) persistence simplification.

Labeling. Next, we apply k-means clustering to generate label correspondences across the simplified Morse complexes (Figs. 4a-b) and compare such a strategy against the one based on mandatory maxima (Figs. 4c-d). The strategy based on k-means and the one based on mandatory maxima are shown to provide slightly different labels for the simplified Morse complexes.

Probabilistic map. We visualize the probabilistic map $\mathcal{P}$ via color blending in Fig. 5. The black contours of the uncertain regions in Fig. 5c visualize the expected 2-cell boundaries. In fact, only a few regions with certainty in Fig. 5b demonstrate

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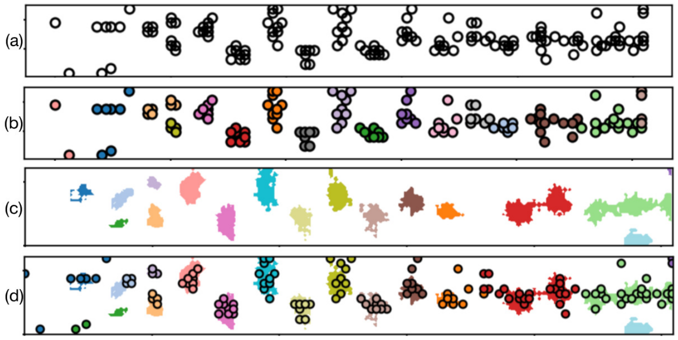

Fig. 4. Kármán vortex street dataset. Labeling with k-means clustering: (a) a scatter plot of maxima across simplified ensemble members; (b) k-means clustering with $k = 17$. Labeling with mandatory maxima: (c) mandatory maxima are shown as colored regions; (d) each maximum is assigned the label of its nearest mandatory maximum.

a gradient flow consistency across all ensemble members. In contrast, the simplified mean field (Fig. 1b) does not capture structural variations of these boundaries.

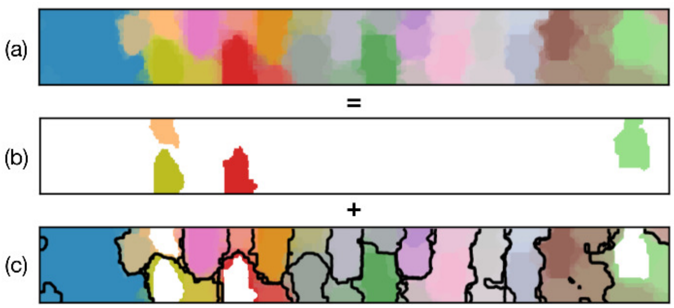

Fig. 5. Kármán vortex street dataset: a probabilistic map $\mathcal{P}$ is visualized using color blending in (a), highlighting regions with certainty (b) and uncertainty (c).

We further explore the entropy-based visualization of $\mathcal{P}$ in Fig. 6. As uniform probability yields maximum uncertainty and therefore entropy, interactively queried locations $x = 1, 2$, and 3 all exhibit a relatively large amount of uncertainty.

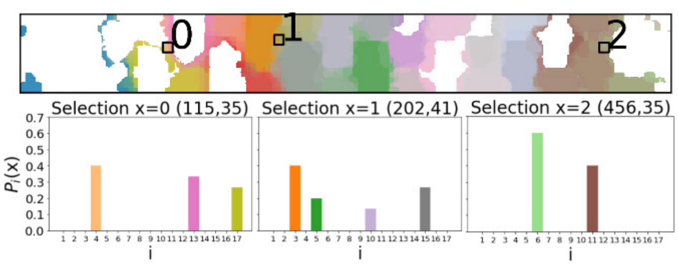

Fig. 6. Kármán vortex street dataset: $\mathcal{P}$ is visualized with point-wise entropy thresholded at ≥ 0.9 and interactive queries.

Significance map. Figs. 7a and b visualize the significance map $\mathcal{I}$ via its point-wise mean and patch-wise entropy, respectively. In particular, the yellow regions in Fig. 7(a) enclose points near the cylinder with the highest persistence across ensembles. Furthermore, the entropy-based visualization quantifies the randomness of the significance values and highlights the spatial uncertainty of 2-cell boundaries. The yellow regions immediately to the right of the cylinder in Figs. 7b are shown to exhibit the highest level of variability.

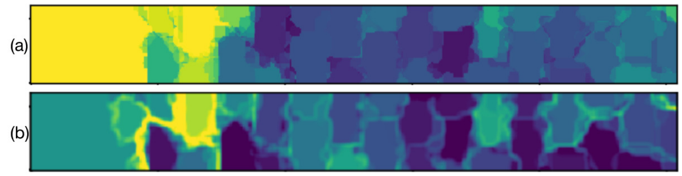

Fig. 7. Kármán vortex street dataset: $\mathcal{I}$ is visualized via its point-wise mean (a) and patch-wise entropy (b).

References

[1] S. Popine, "Gerris: A tree-based adaptive solver for the incompressible Euler equations in complex geometries," *Journal of Computational Physics*, vol. 190, no. 2, pp. 572–600, 2003.

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