Appendix

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A tabulation is provided for the probability density function(PDF) of a random variable modeling the level crossing on an edge, when data is corrupted with an independent uniform noise. The expressions for the PDF are provided over the domain $(-\infty, \infty)$. However, we consider functions only falling in the range [0,1] in the marching cubes framework. Table covers the non-overlapping, overlapping and contained interval cases as explained in the paper. Suppose we have a knowledge of the parameters μ_1, δ_1 and μ_2, δ_2 corresponding to the random variables X_1 and X_2, respectively, and an isovalue c. This sets up a parallelogram PQRS, representing the joint distribution of the random variables Z_1 and Z_2, as shown in the paper (Note down the order of vertices P,Q,R and S). For all cases, the assumption is that $\mu_2 > \mu_1$ without a loss of generality. The slopes of the corners of the parallelogram are given as follows (Slope is indicated using vertex name itself).

$$\mathrm{P} = \frac{\mathrm{c} - \mu_1 - \delta_1}{\mu_2 - \mu_1 + \delta_2 - \delta_1} \qquad \mathrm{Q} = \frac{\mathrm{c} - \mu_1 + \delta_1}{\mu_2 - \mu_1 + \delta_2 + \delta_1}$$

$$\mathrm{S} = \frac{\mathrm{c} - \mu_1 - \delta_1}{\mu_2 - \mu_1 - \delta_2 - \delta_1} \qquad \mathrm{R} = \frac{\mathrm{c} - \mu_1 + \delta_1}{\mu_2 - \mu_1 + \delta_1 - \delta_2}$$

Slope ordering plays an important role in determining PDF for most cases. Also, taking into account the fact that slanted sides of the parallelogram are at 45° angle, we can prove whether a particular slope ordering may or may not take place for the given isovalue case. Using the distribution parameters and c, the following variables, which are part of the final expression for the PDF, are precomputed.

$$\mathrm{f}_1 = (\mu_1 + \delta_1 - \mathrm{c})^2$$
$$\mathrm{f}_2 = (\mu_1 - \delta_1 - \mathrm{c})^2$$
$$\mathrm{f}_3 = (\mu_2 + \delta_2 - \mathrm{c})^2$$
$$\mathrm{f}_4 = (\mu_2 - \delta_2 - \mathrm{c})^2$$

Each table entry is represented as a pair x, y, which represents a function $\frac{\mathrm{x}\mathrm{z}^2 + \mathrm{y}(1-\mathrm{z})^2}{8\delta_1\delta_2\mathrm{z}^2(1-\mathrm{z})^2}$. x and y are functions of the parameters $\mu_1, \delta_1, \mu_2, \delta_2$ and c. Entry 0 represents a $0, 0$ pair. For a vertex order ABCD, a piecewise density function is represented by 5 pieces $\mathrm{P}_0, \mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3, \mathrm{P}_4$ such that $-\infty < \mathrm{P}_0 \leq \mathrm{A} \leq \mathrm{P}_1 \leq \mathrm{B} \leq \mathrm{P}_2 \leq \mathrm{C} \leq \mathrm{P}_3 \leq \mathrm{D} \leq \mathrm{P}_4 < \infty$.

1) Non-Overlapping Intervals: $(\mu_1 + \delta_1 \le \mu_2 - \delta_2)$

Order	Probability Density Function				
$\mu_1 - \delta_1 \le c \le \mu_1 + \delta_1$					
SPQR	0	$-f_4, f_1$	$4\delta_2(\mu_2 - c), 0$	$-f_4, f_2$	0
$\mu_1 + \delta_1 < c \le \mu_2 - \delta_2$					
PQSR	0	$f_3, -f_1$	$0, 4\delta_1(c - \mu_1)$	$-f_4, f_2$	0
PSQR	0	$f_3, -f_1$	$4\delta_2(\mu_2 - c), 0$	$-f_4, f_2$	0
$\mu_2 - \delta_2 < c \le \mu_2 + \delta_2$					
PQRS	0	$f_3, -f_1$	$0, 4\delta_1(c - \mu_1)$	$f_4, -f_1$	0

2) Overlapping Intervals: $(\mu_1 - \delta_1 < \mu_2 - \delta_2 \le \mu_1 + \delta_1 < \mu_2 + \delta_2)$

Order	Probability Density Function				
$\mu_1 - \delta_1 \le c \le \mu_2 - \delta_2$					
PQRS	$-f_4, f_1$	$4\delta_2(\mu_2 - c), 0$	$-f_4, f_2$	0	$-f_4, f_1$
$\mu_2 - \delta_2 < c \le \mu_1 + \delta_1$					
PSQR	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_3, f_1	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1
PQSR	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_4, f_2	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1
$\mu_1 + \delta_1 < c \le \mu_2 + \delta_2$					
SPQR	$f_4, -f_1$	0	$f_3, -f_1$	$0, 4\delta_1(c - \mu_1)$	$f_4, -f_1$

3a) Contained Intervals: $(\mu_1 - \delta_1 \le \mu_2 - \delta_2 \le \mu_2 + \delta_2 \le \mu_1 + \delta_1)$

Order	Probability Density Function				
$\mu_1 - \delta_1 \le c \le \mu_2 - \delta_2$					
QRSP	$4\delta_2(\mu_2 - c), 0$	$-f_4, f_2$	0	$-f_4, f_1$	$4\delta_2(\mu_2 - c), 0$
$\mu_2 - \delta_2 < c \le \mu_2 + \delta_2$					
SQRP	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_3, f_1	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$
QSRP	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_4, f_2	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$
SQPR	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_3, f_1	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_3, f_2	$2((c - \mu_2)^2 + \delta_2^2), 0$
QSPR	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_4, f_2	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_3, f_2	$2((c - \mu_2)^2 + \delta_2^2), 0$
$\mu_2 + \delta_2 < c \le \mu_1 + \delta_1$					
SPQR	$4\delta_2(c - \mu_2), 0$	$-f_3, f_1$	0	$-f_3, f_2$	$4\delta_2(c - \mu_2), 0$

3b) Contained Intervals: $(\mu_2 - \delta_2 \le \mu_1 - \delta_1 \le \mu_1 + \delta_1 \le \mu_2 + \delta_2)$

Order	Probability Density Function				
$\mu_2 - \delta_2 \le c \le \mu_1 - \delta_1$					
PQRS	$0, 4\delta_1(\mu_1 - c)$	$f_3, -f_2$	0	$f_4, -f_2$	$0, 4\delta_1(\mu_1 - c)$
$\mu_1 - \delta_1 < c \le \mu_1 + \delta_1$					
RPSQ	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_3, f_1	$0, 2((c - \mu_1)^2 + \delta_1^2)$
PRSQ	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_3, f_2	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_3, f_1	$0, 2((c - \mu_1)^2 + \delta_1^2)$
RPQS	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_4, f_1	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_4, f_2	$0, 2((c - \mu_1)^2 + \delta_1^2)$
PRQS	$0, 2((c - \mu_1)^2 + \delta_1^2)$	f_3, f_2	$2((c - \mu_2)^2 + \delta_2^2), 0$	f_4, f_2	$0, 2((c - \mu_1)^2 + \delta_1^2)$
$\mu_1 + \delta_1 < c \le \mu_2 + \delta_2$					
RSPQ	$0, 4\delta_1(c - \mu_1)$	$f_4, -f_1$	0	$f_3, -f_1$	$0, 4\delta_1(c - \mu_1)$