VIS 2020

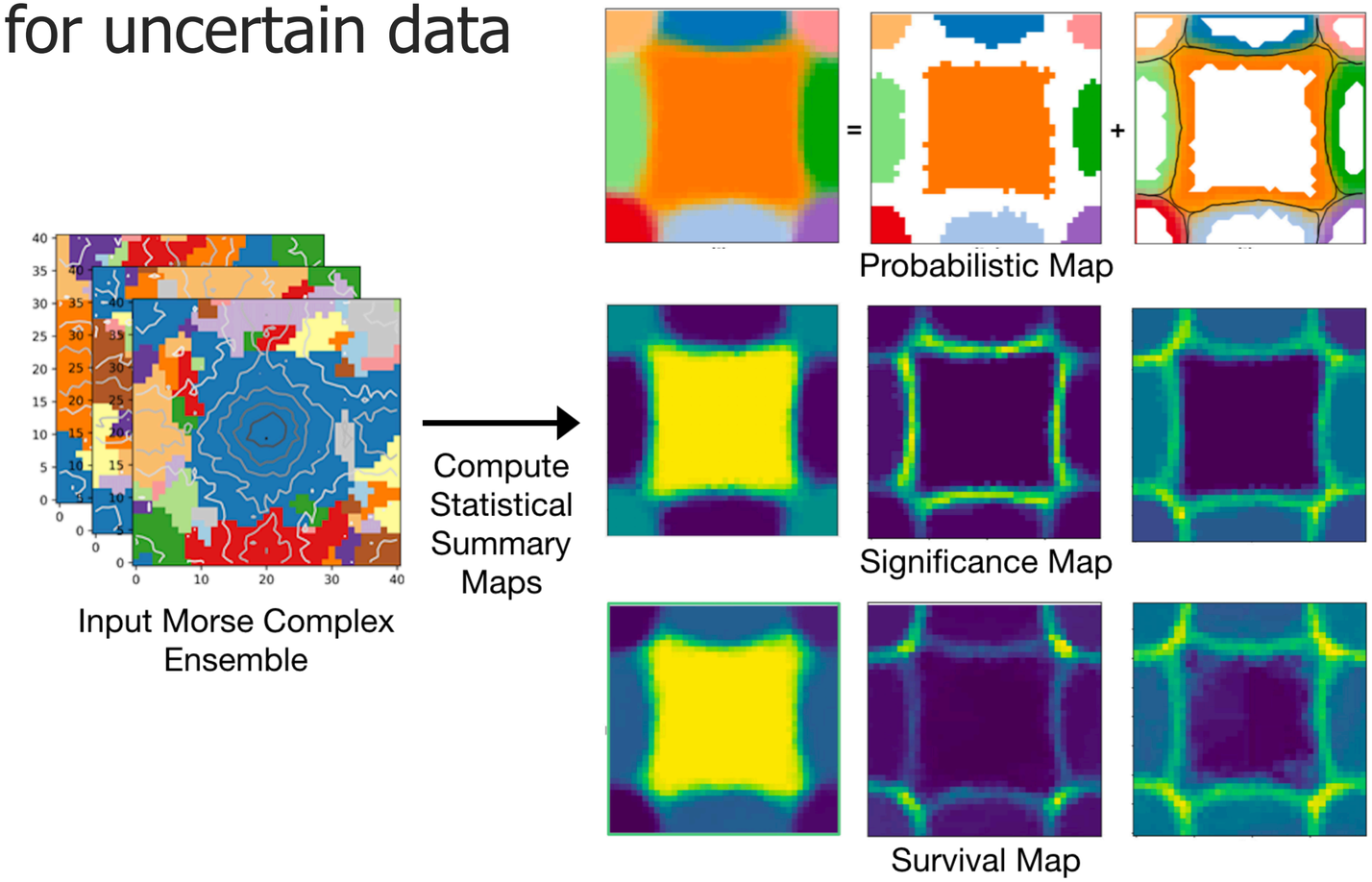
Uncertainty Visualization of 2D Morse Complex Ensembles Using Statistical Summary Maps

Tushar M. Athawale, Dan Maljovec, Lin Yan,
Chris R. Johnson, Valerio Pascucci, and Bei Wang

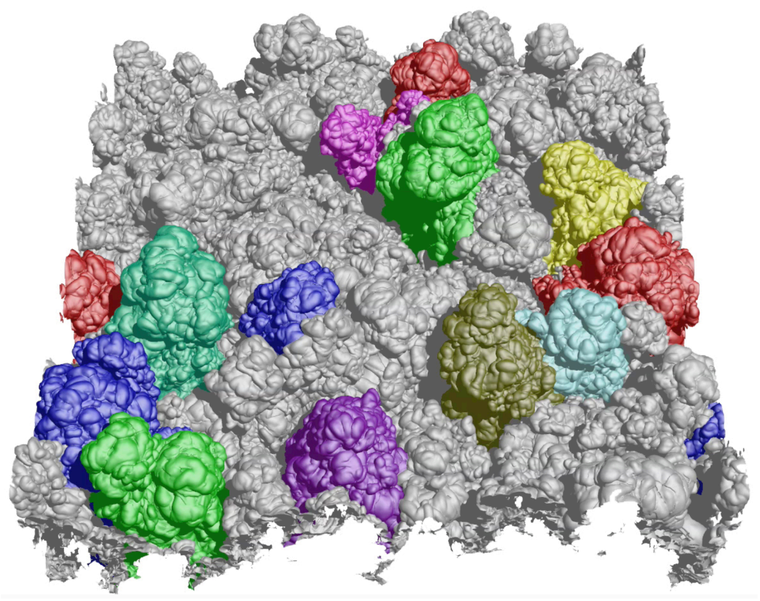
Scientific Computing & Imaging (SCI) Institute, University of Utah

Outline

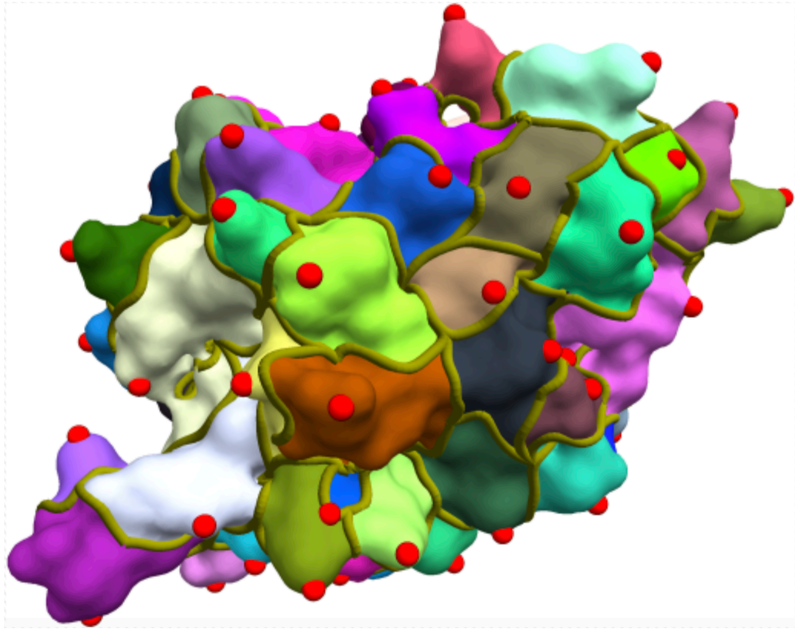
- Topology-based visualizations for uncertain data
  - Morse complexes
  - Related work
- Statistical summary maps
  - Uncertainty visualization of Morse complexes
- Results, conclusion, and future work



Morse Complexes



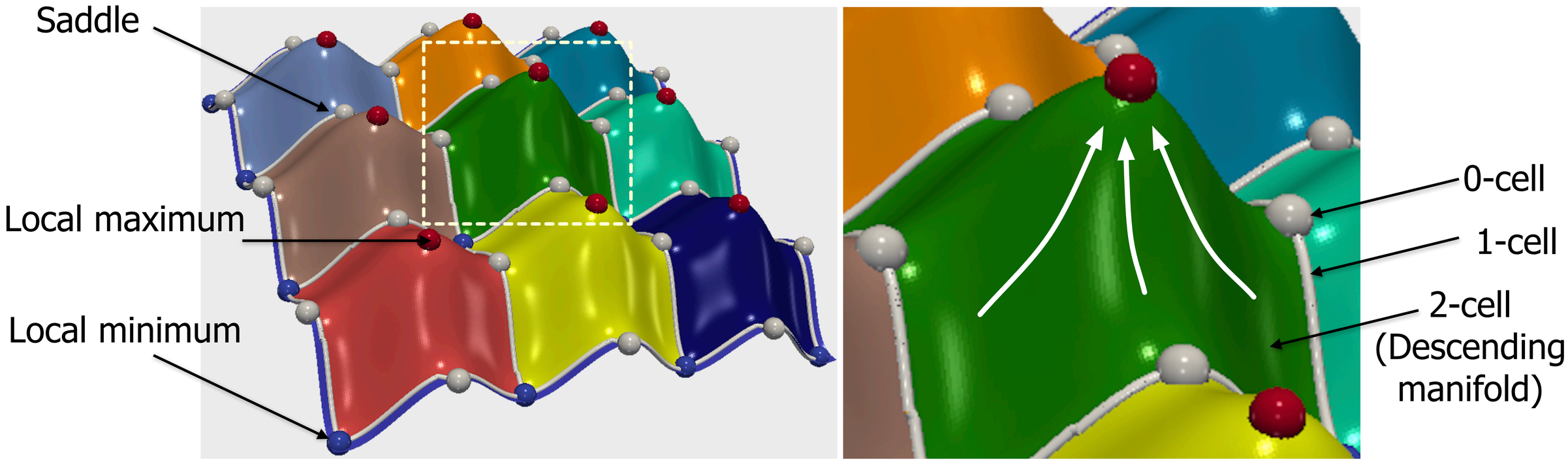
Understanding structure of turbulent mixing layers [Laney et al. 2006]



Segmenting molecular surfaces
[Natarajan et al., 2006]
[Shivashankar et al., 2012]

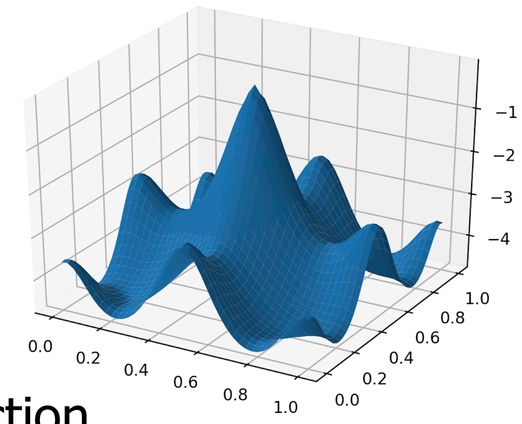
Morse Complexes

Gradient-based topological descriptors of scalar fields



(Visualization software: The Topology Toolkit (TTK) [Tierny et al., 2017])

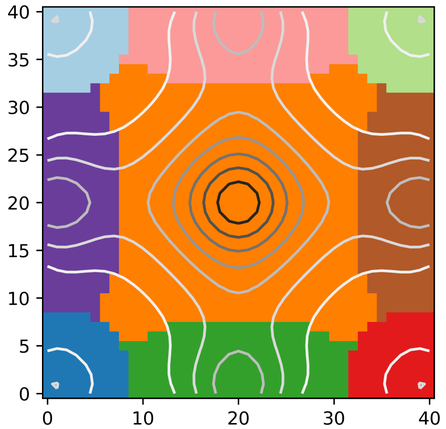
Effect of Noise on Morse Complexes



Ackley function
[Ackley, 1987]

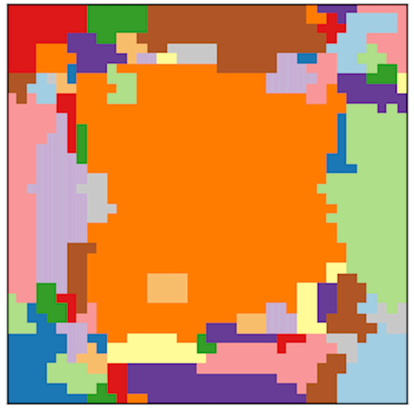
Morse complex extraction

Ground truth Morse complex

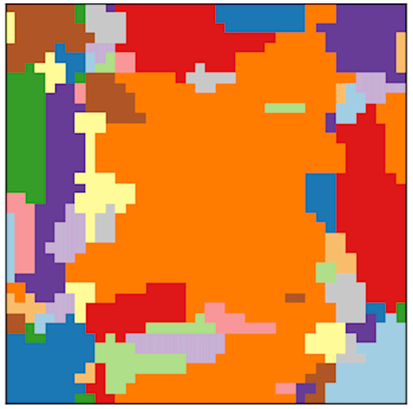


Mix noise and
extract Morse complex

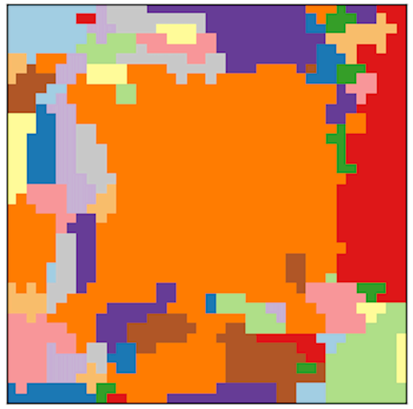
Ensemble member 1



Ensemble member 2



Ensemble member 3



Related Work: Topological Uncertainty Visualization

- Uncertainty visualization of critical points for uncertain scalar field [Mihai and Westermann, 2014; Günther et al., 2014; Liebmann and Scheuermann, 2016; Favelier et al., 2019] and multifield data [Huettenberger et al., 2013]
- Uncertainty visualization of gradient flows for uncertain scalar field [Otto et al., 2010, 2011; Bhatia et al., 2012; Pfaffelmoser et al., 2013] and multifield data [Nagaraj et al., 2011]
- Uncertainty visualization of contour trees [Kraus, 2010; Wu and Zhang, 2012; Zhang et al., 2015; Yan et al. 2020, Lohfink et al., 2020]
- Positional likelihood visualization of Morse complexes [Thompson et al., 2011]

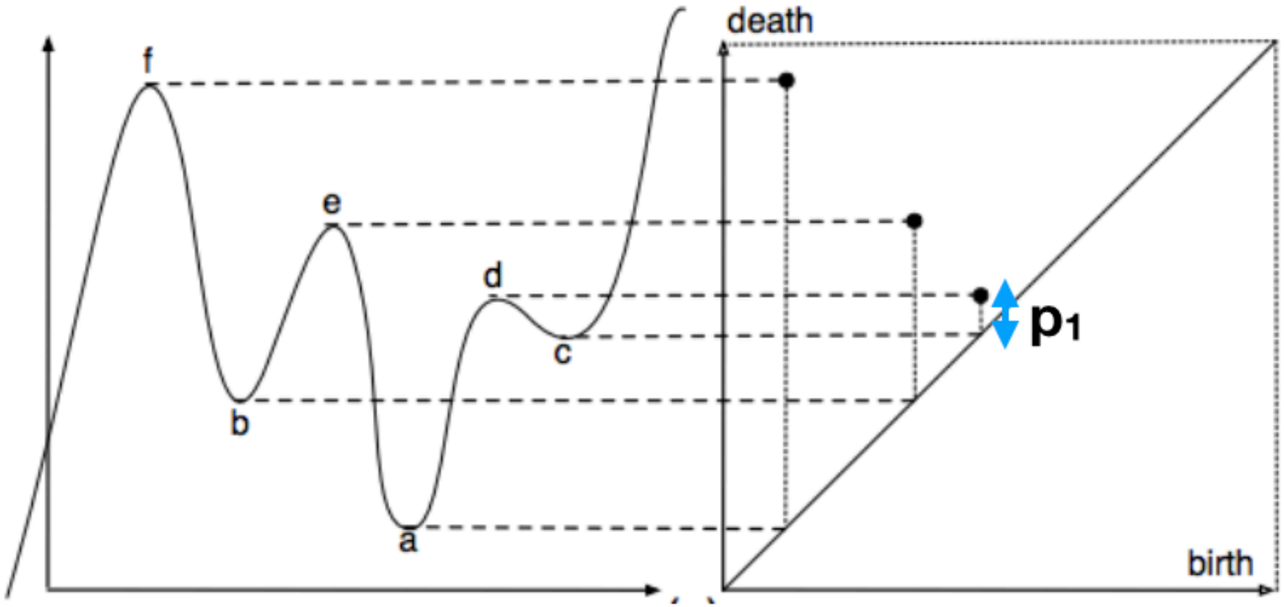
Statistical Summary Maps

Visualize commonalities and differences in Morse complexes for ensembles via the study of variability in gradient flows and critical points:

- Probabilistic maps
- Significance maps
- Survival maps

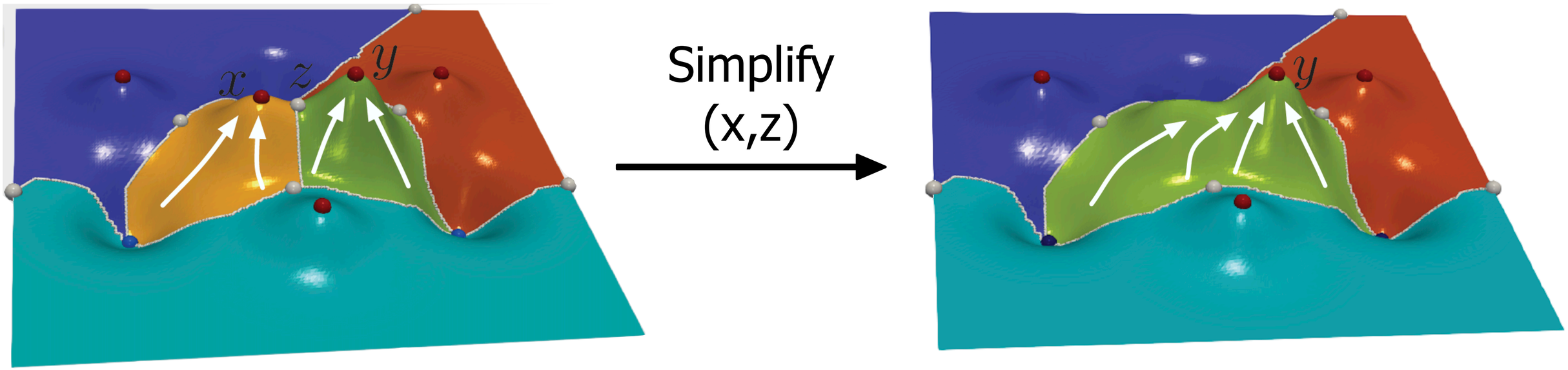
Persistence

A tool to quantify significance of topological features



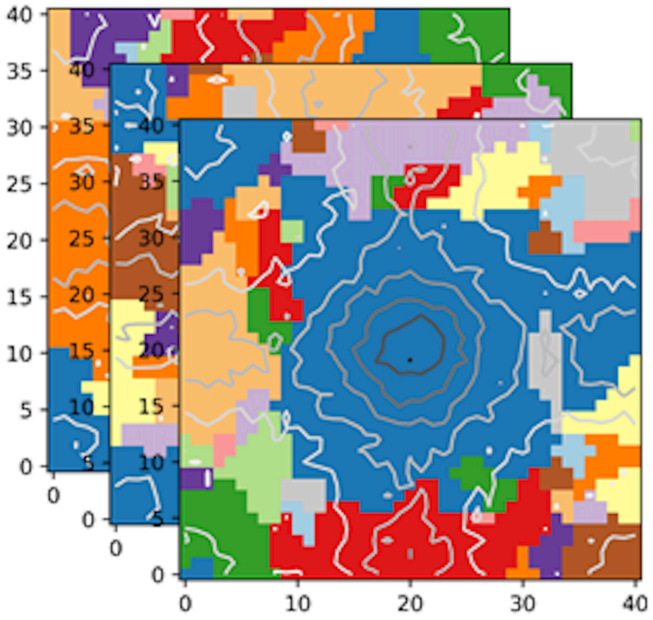
Topological Simplification [Edelsbrunner et al., 2003]

A tool for de-noising of a scalar field



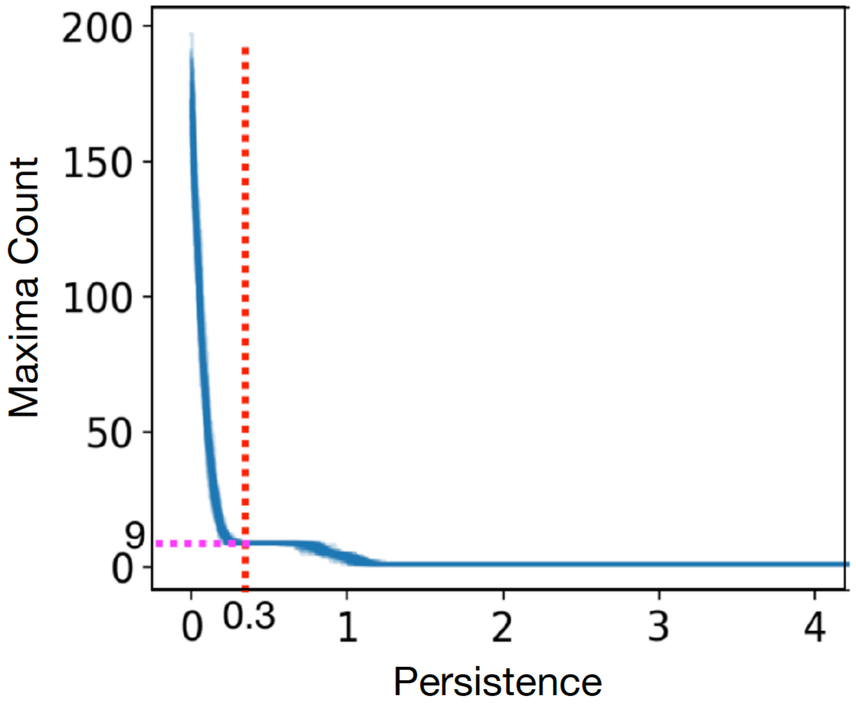
Probabilistic Map

Step 1: Topological Simplification

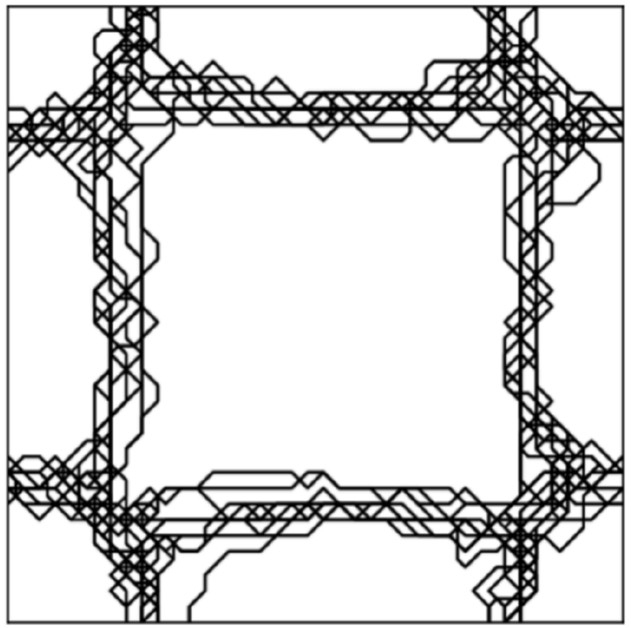


Input Morse complex ensemble

Persistence graph
[Gerber et al., 2010]

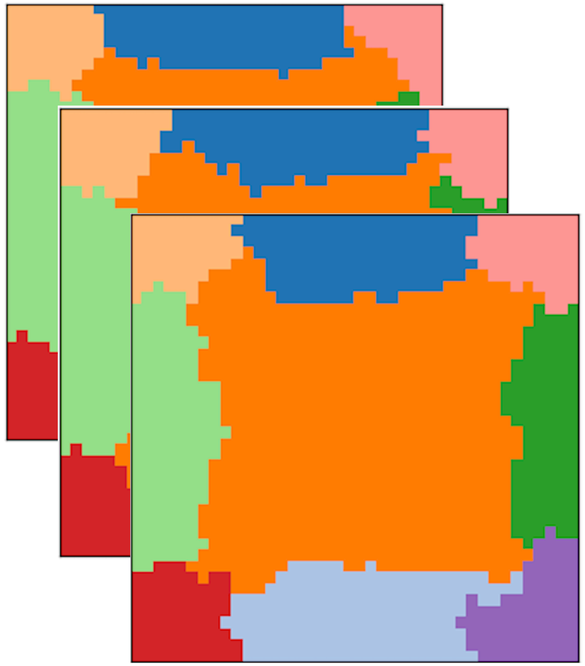


Spaghetti plot of Morse complexes for simplification level 0.3



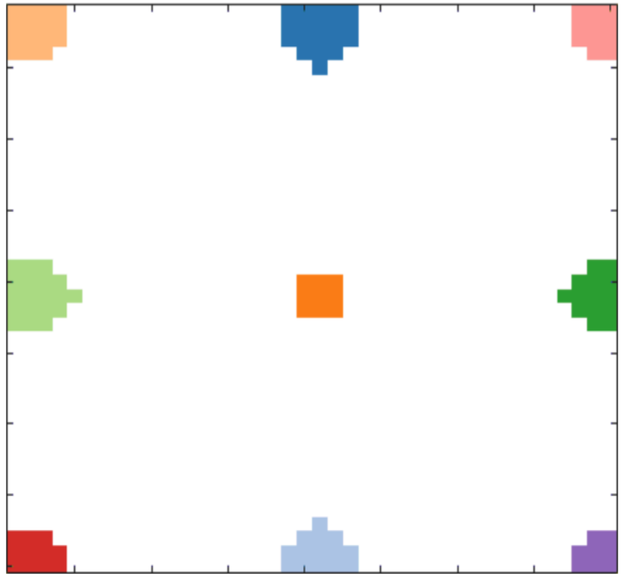
Maxima count = 9

Step 2: Labeling of Local Maxima

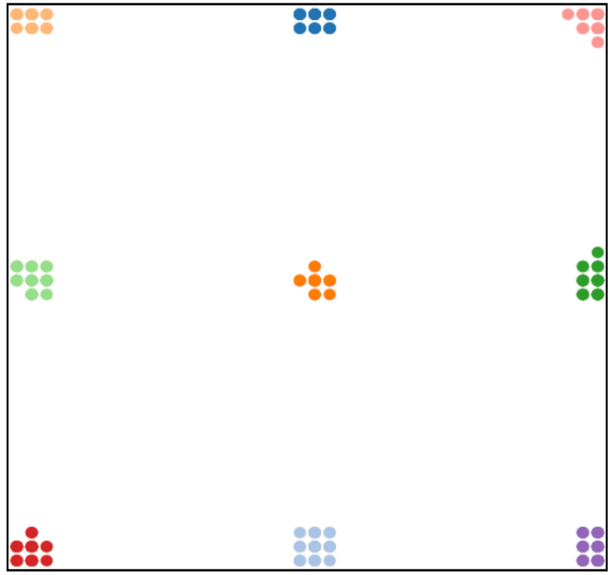


Simplified ensemble
with #2-cells = 9

Mandatory maxima
[Günther et al., 2014]



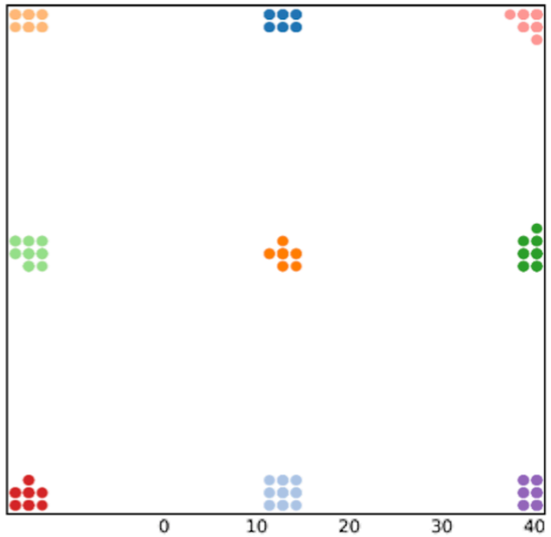
Nearest mandatory maxima



$l \in [1,\ldots,9]$

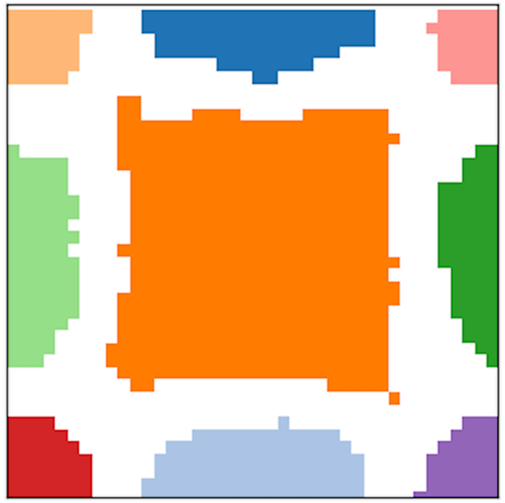
or k-means with k = 9
or Morse mapping [Reininghaus et al., 2012]

Step 3: Visualization



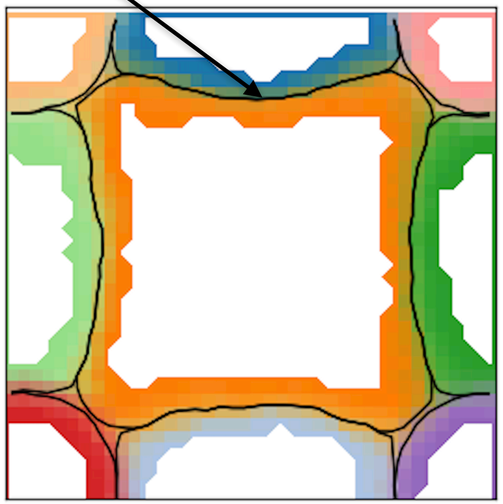
Labeled maxima

Compute per-pixel gradient destination probability



100% agreement (certainty) regions

0.5 probability of flowing to a single label (expected Morse complex)



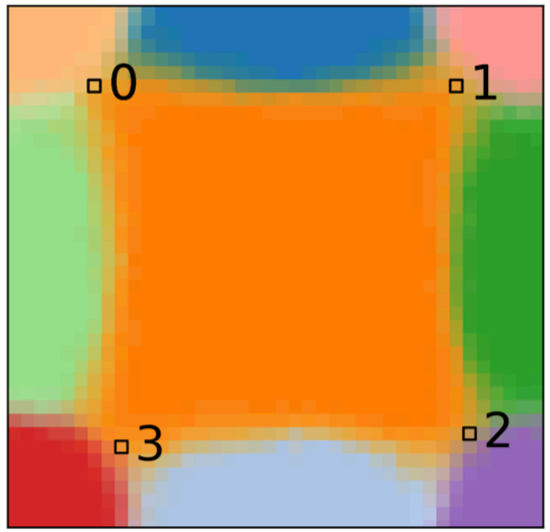
Uncertainty regions

$$\text{Expected color} = \sum_i c_i p_i$$

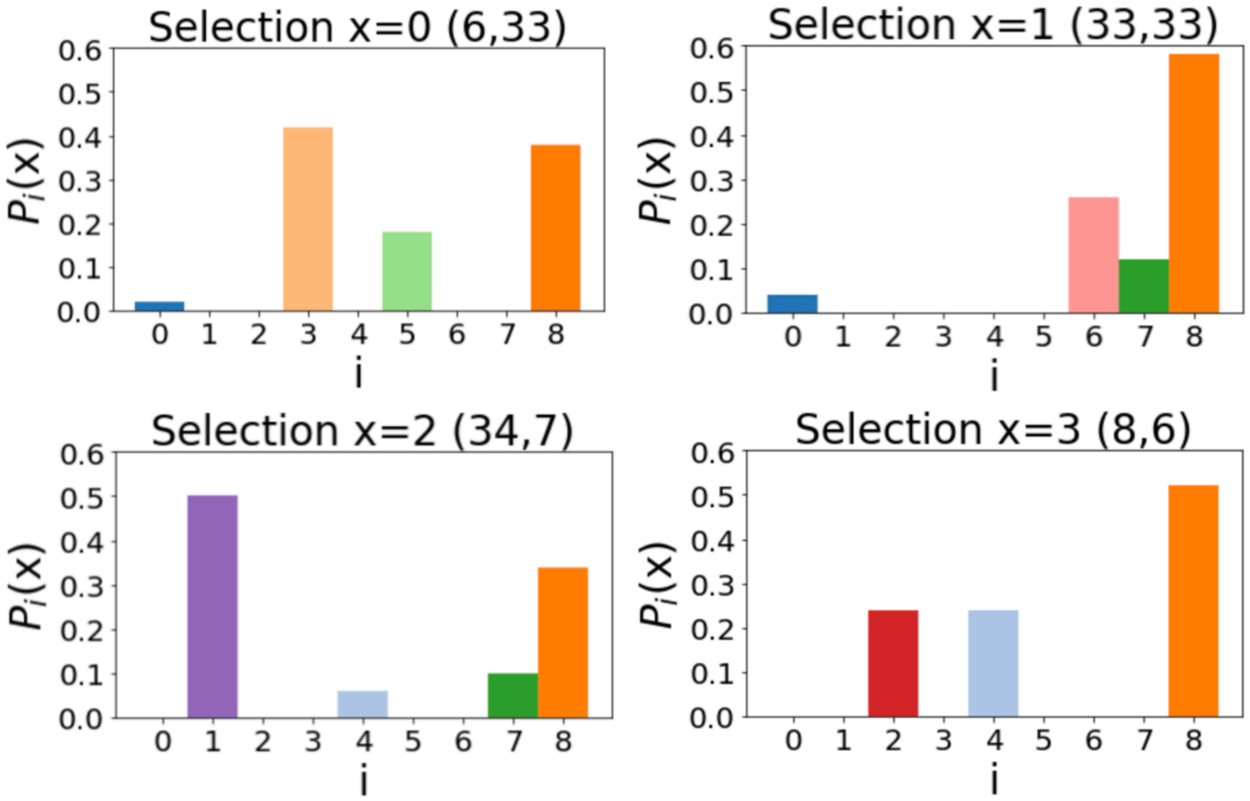
c_i: Color denoting a label

p_i: Probability of gradient flow terminating in label c_i

Interactive PDF Queries



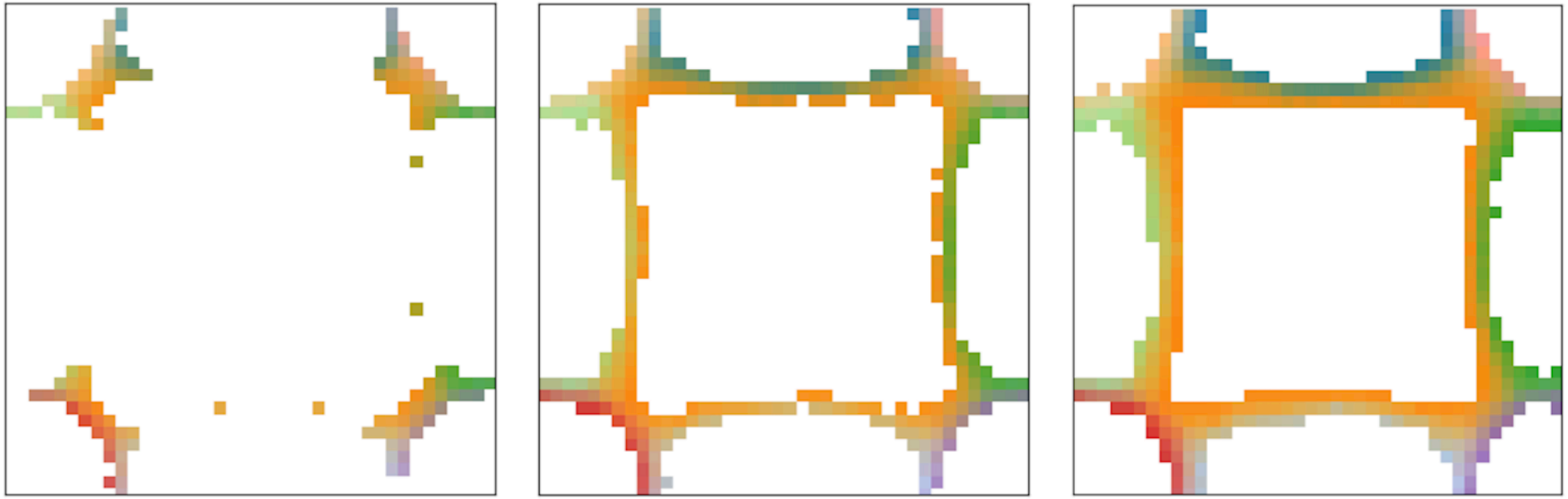
Probabilistic map



[K. Potter, R. M. Kirby, D. Xiu, and C. R. Johnson; Interactive visualization of probability and cumulative density functions; 2011]

Entropy-based Uncertainty Exploration

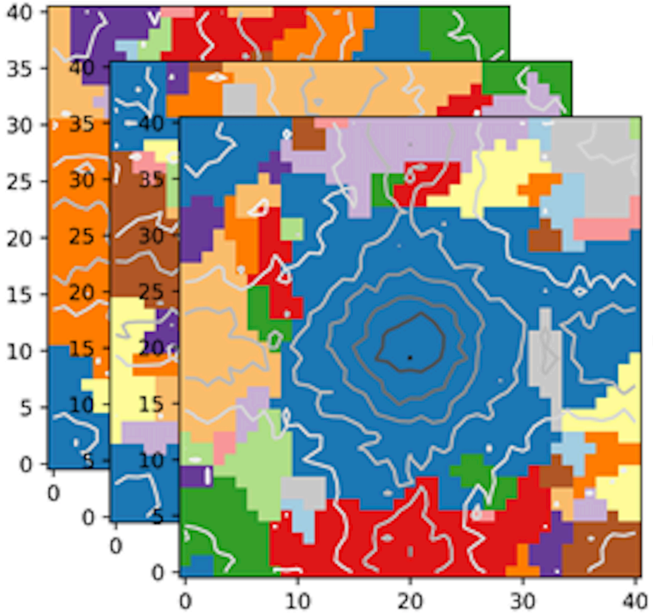
The uniform distribution yields maximum uncertainty, and therefore, maximum entropy.



(a) Entropy >= 1 (b) Entropy >= 0.75 (c) Entropy >= 0.5

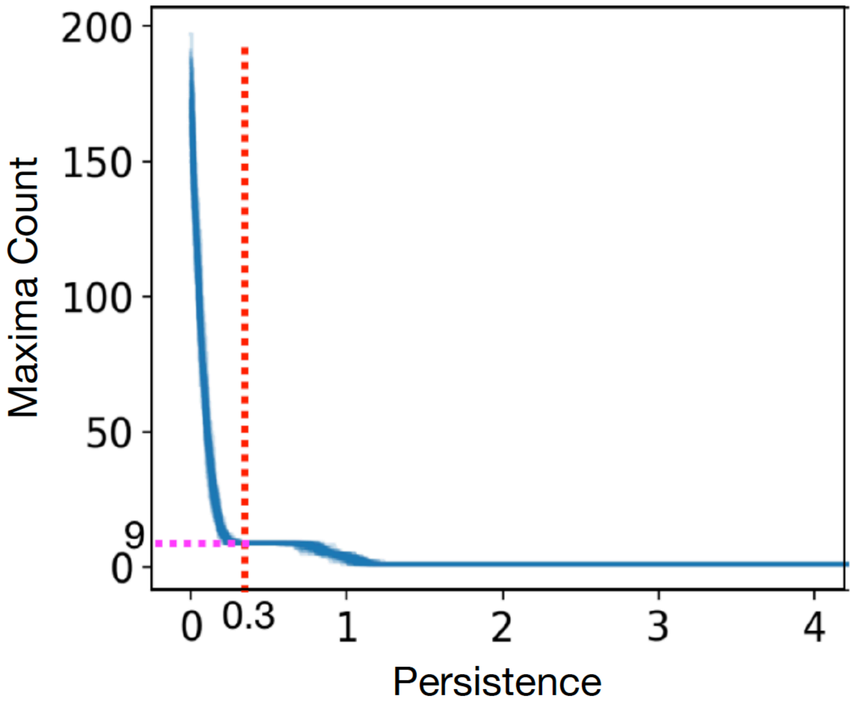
Significance Map

Step 1: Topological Simplification

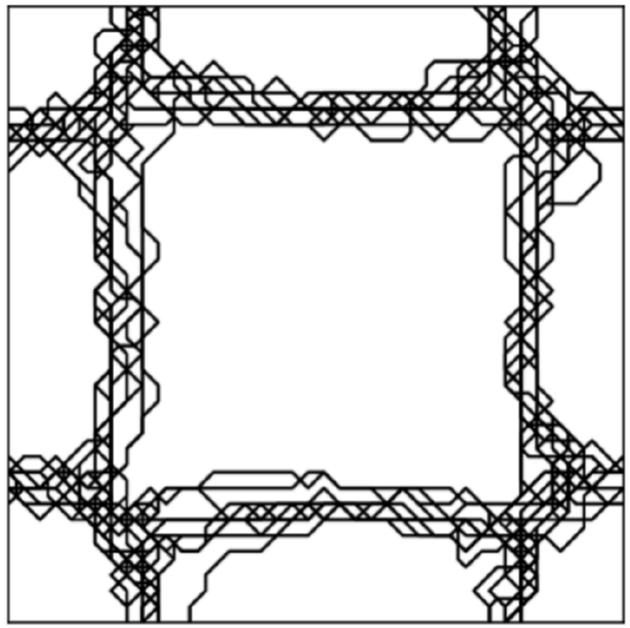


Input Morse complex ensemble

Persistence graph
[Gerber et al., 2010]

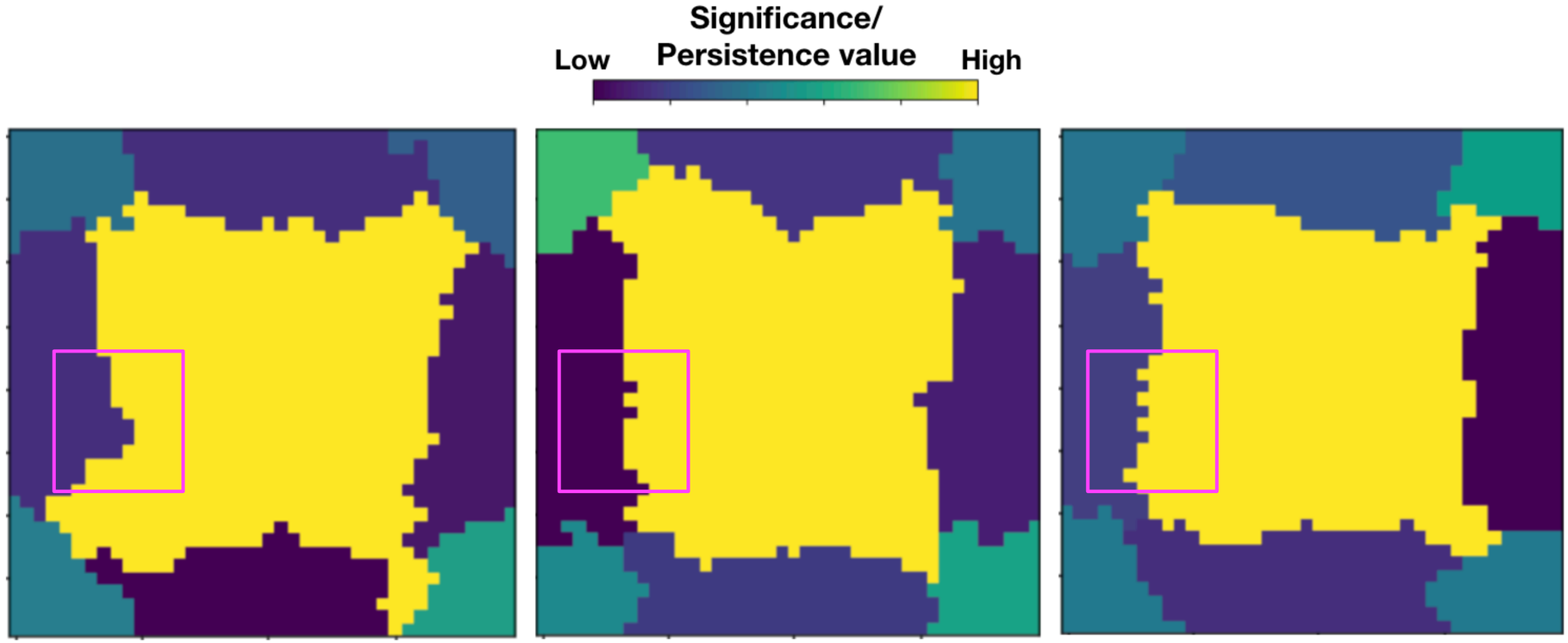


Spaghetti plot of Morse complexes for simplification level 0.3



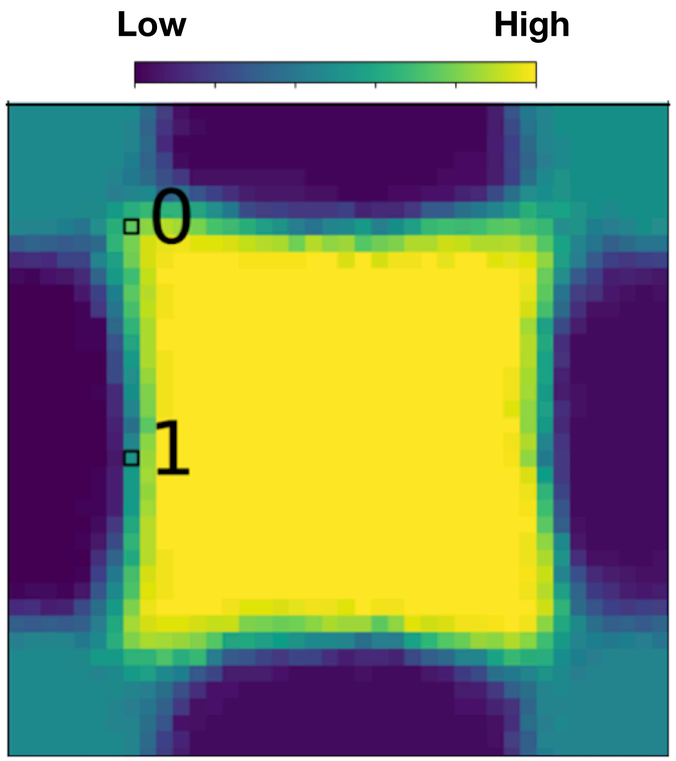
Maxima count = 9

Step 2: Assign Persistence to Each 2-Cell

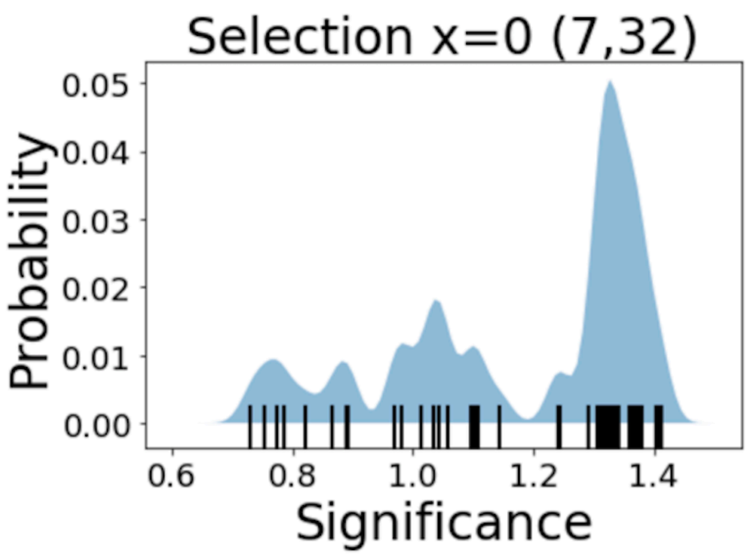


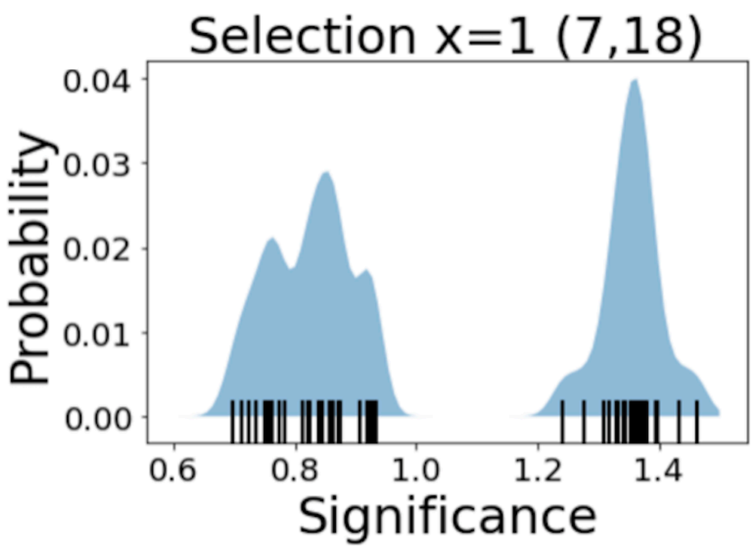
Gradient flow originating at pixels may terminate into feature with high/low persistence, as indicated in magenta boxes

Step 3: Visualization

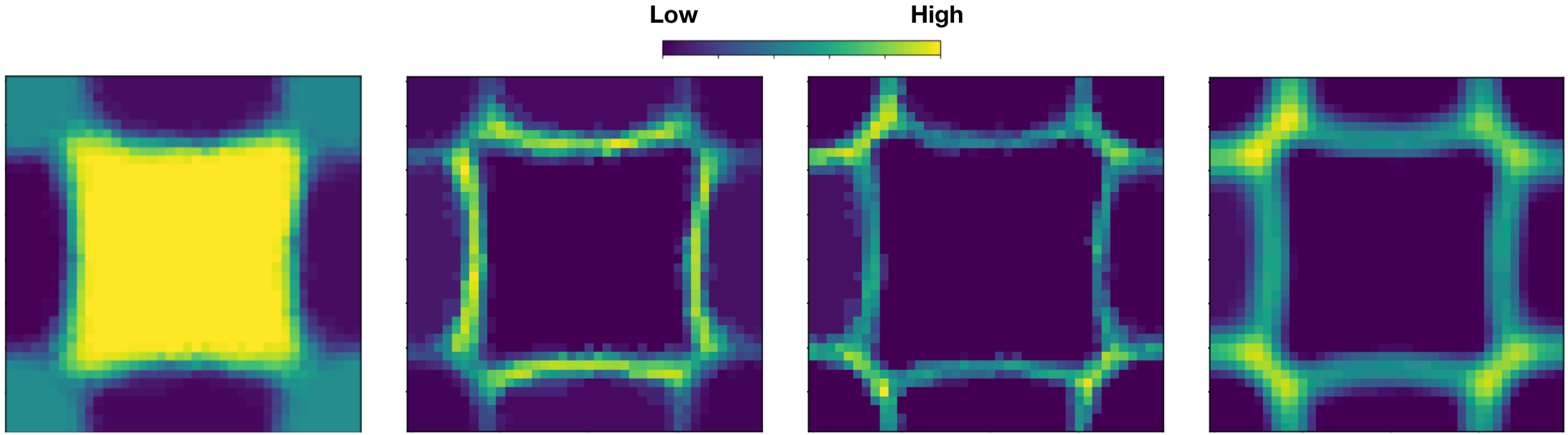


Point-wise mean significance





Variance and Entropy Visualizations



Point-wise mean significance

Point-wise variance

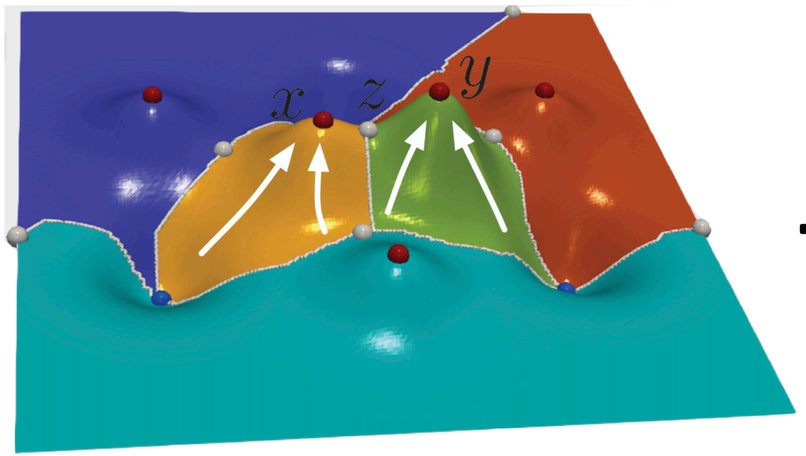
Point-wise entropy

Patch-wise entropy (5x5 window)

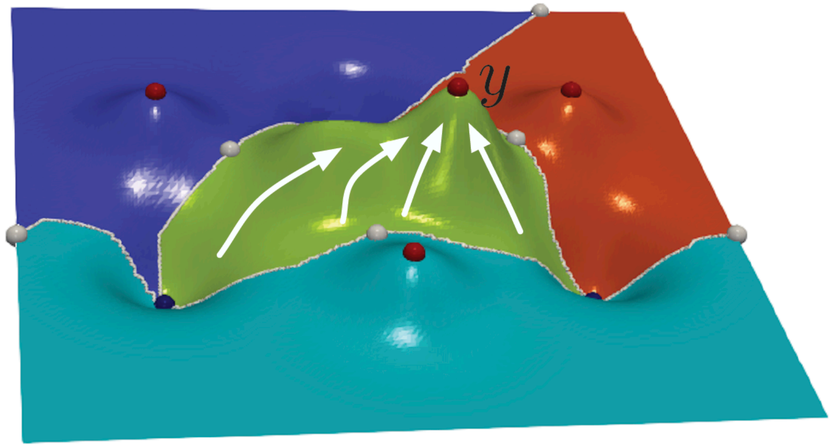
Survival Map

Core Step: Computing Local Gradient Survival

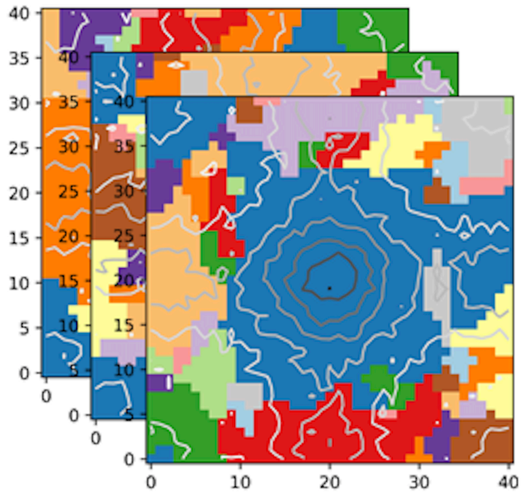
Increment survival of a green 2-cell by the persistence value of orange 2-cell since the gradients in the green cell survived the local topological simplification



Simplify
(x,z)



Survival Map Computation



Input Morse complex ensemble

Pick single member

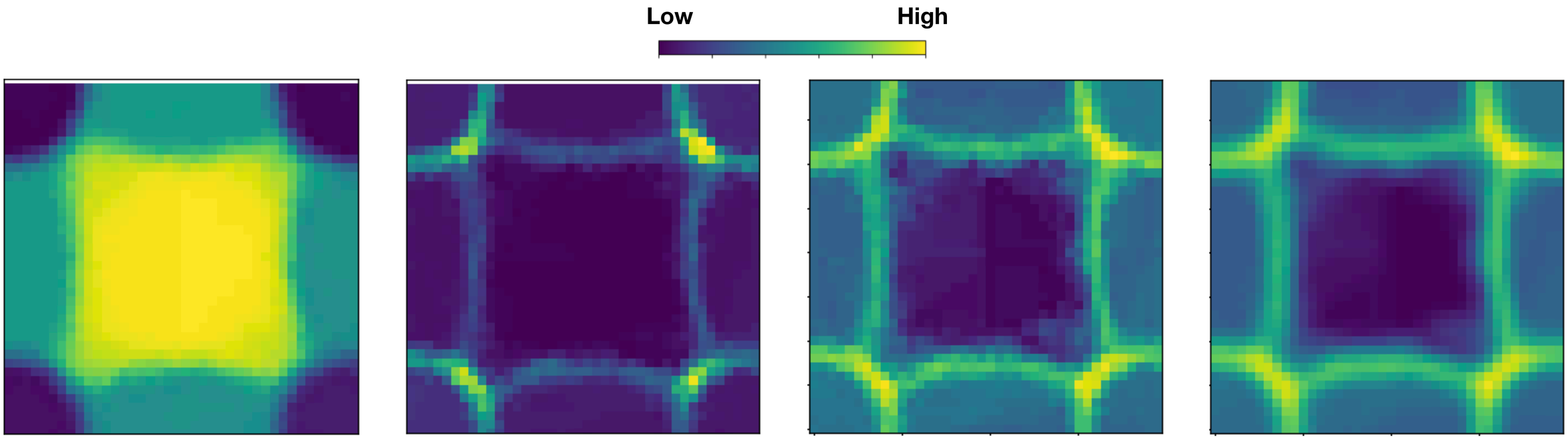
Step1: Compute feature persistences and sort in increasing order

$$\lambda_1 < \cdots < \lambda_n$$

Step 2: Apply topological simplification and compute local survival for each λ_i

Repeat Steps 1 and 2 for all members

Visualization



Point-wise mean survival

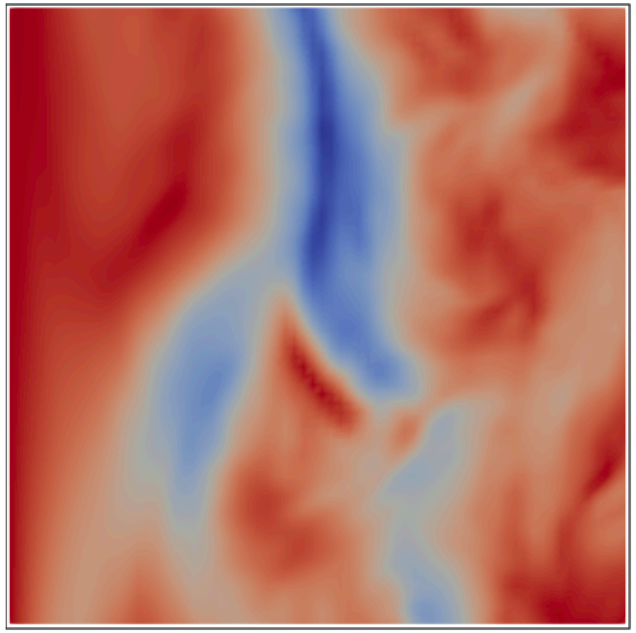
Point-wise variance

Point-wise entropy

Patch-wise entropy (3x3 window)

Results, Conclusion, and Future Work

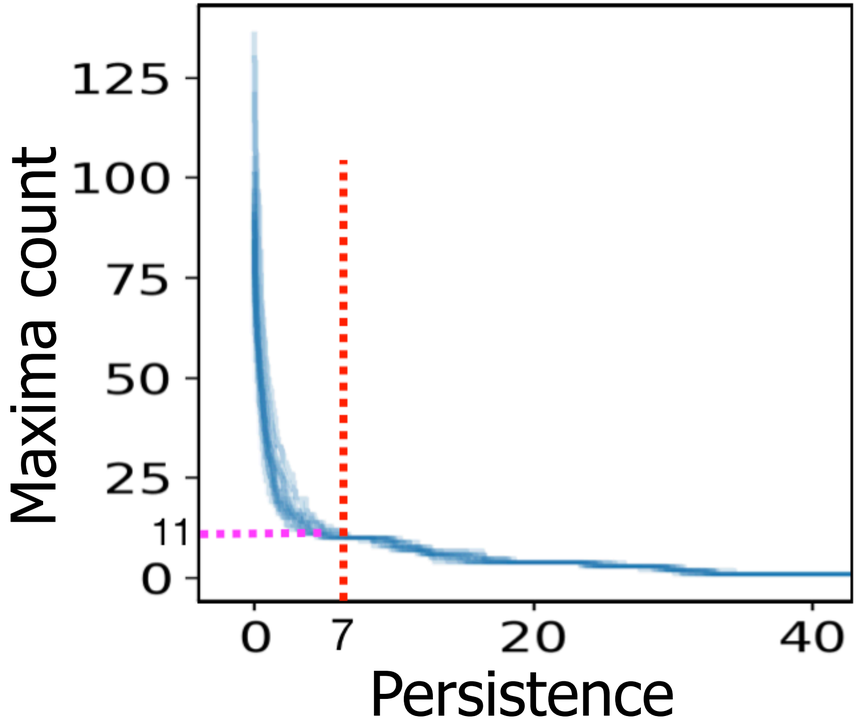
Wind Dataset



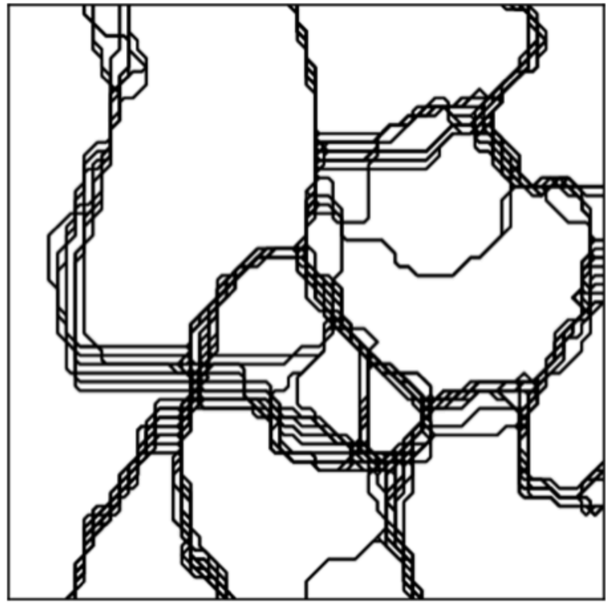
Color-mapped mean velocity magnitude field

(#Ensemble members = 15)

Persistence graphs



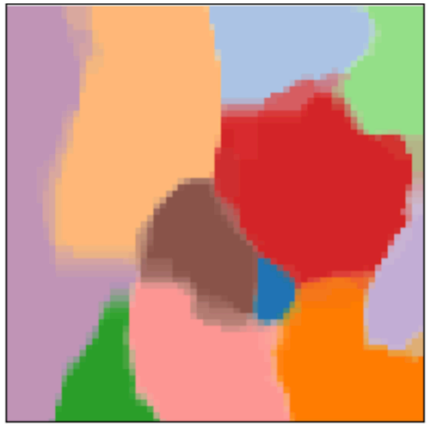
Morse complex spaghetti plot



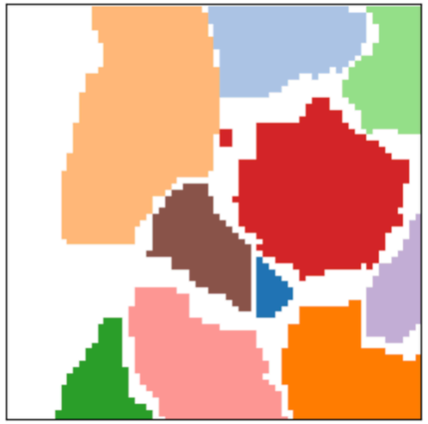
Courtesy: http://iridl.ldeo.columbia.edu/SOURCES/.ECMWF/.S2S/

Wind Dataset: Probabilistic Map Vs. Mean-field

Probabilistic map



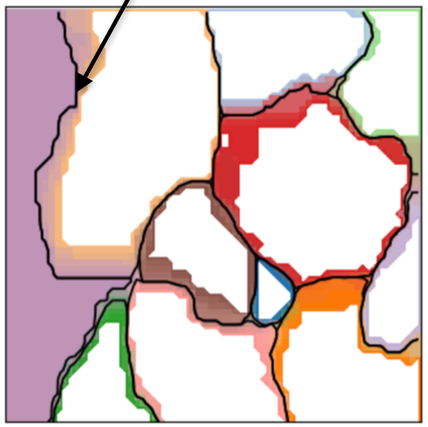
=



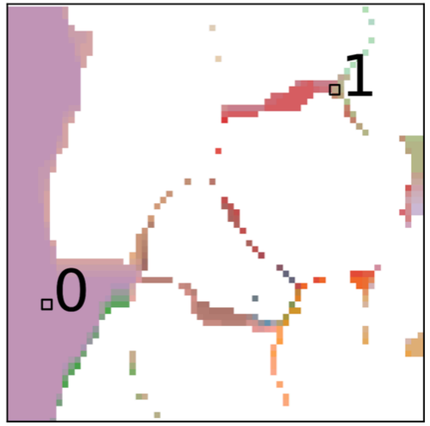
certain regions

+

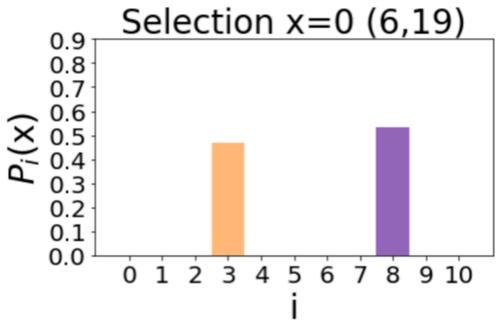
Expected

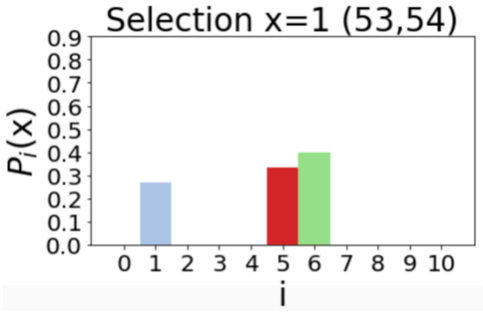


uncertain regions



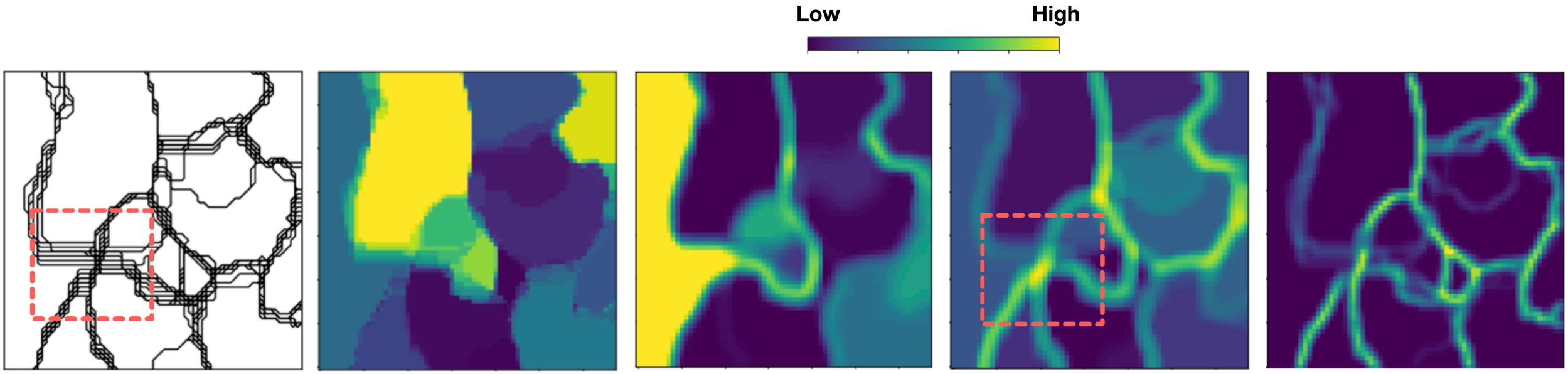
Entropy >= 0.9





Mean-field
Morse complex
(no indication of
spatial uncertainty!)

Wind Dataset: Significance Map



Spaghetti plot

Point-wise mean

Patch-wise variance (3x3 window)

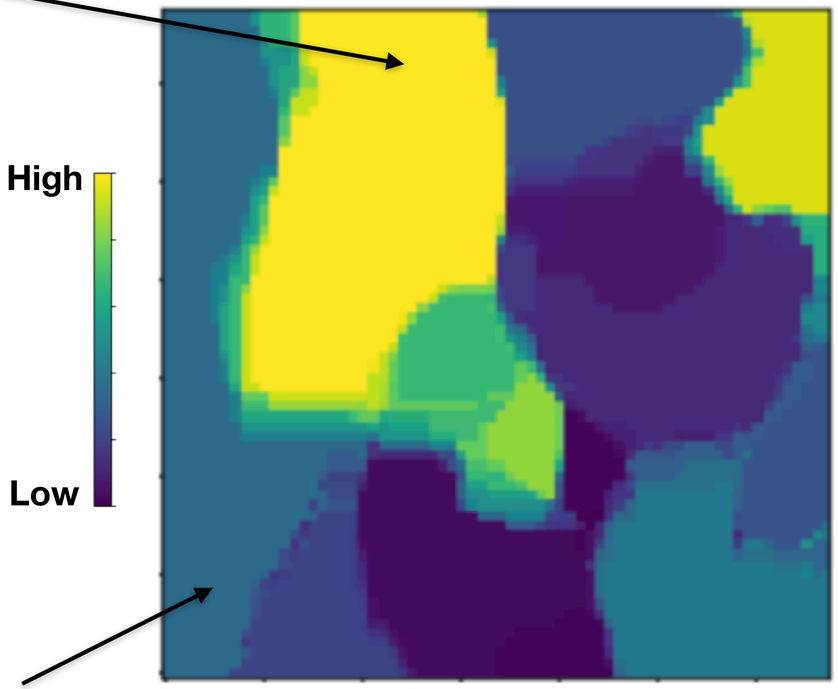
Patch-wise entropy (3x3 window)

Aggregate segmentation [Thompson et al., 2011]

Wind Dataset: Significance vs. Survival Map

High-persistence feature

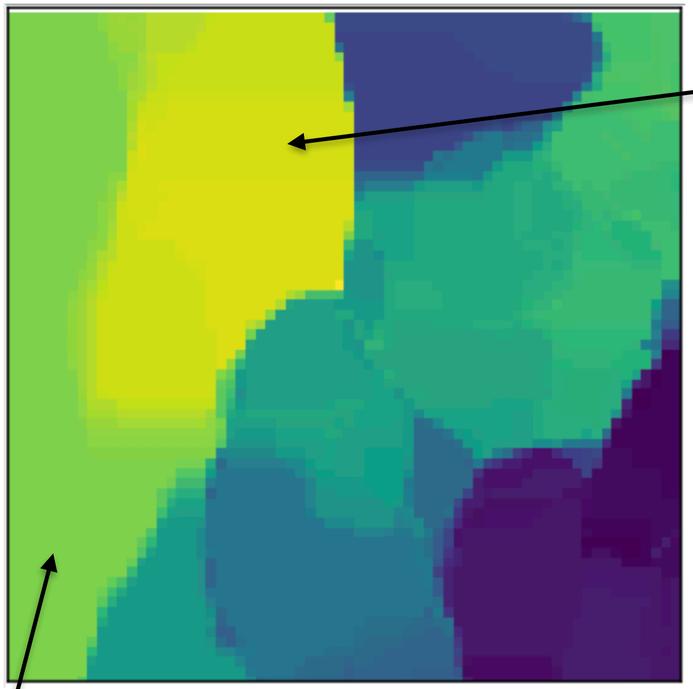
Fixed simplification scale



Low-persistence feature

Point-wise mean significance

All simplification scales



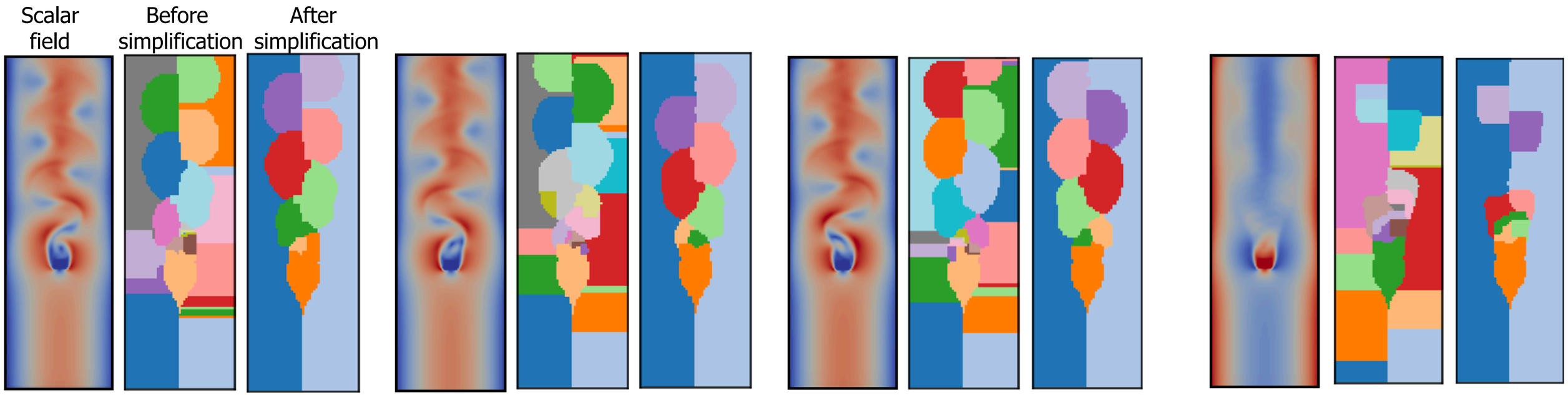
High-persistence feature

High-persistence feature

Point-wise mean survival

Navier Stokes Simulation Dataset

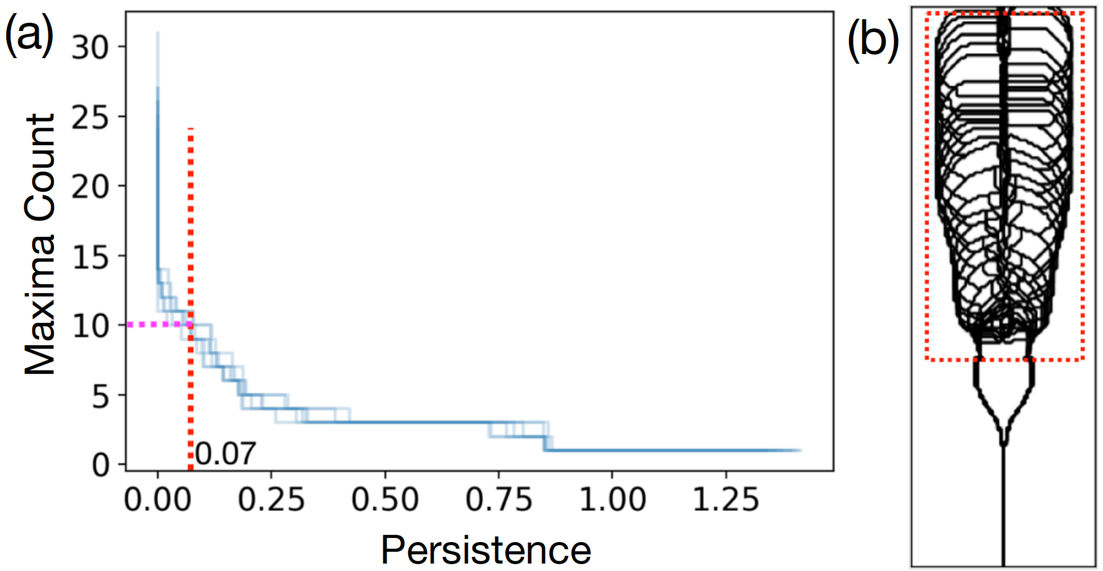
Time-dependent velocity magnitude field



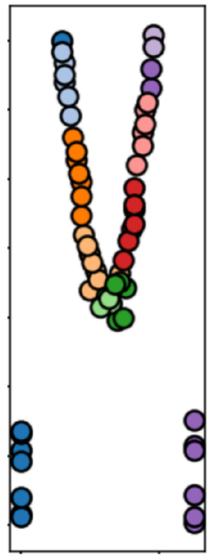
Time step = 60 Time step = 63 Time step = 66 Mean-field

Courtesy: http://tinoweinkauf.net/notes/squarecylinder.html

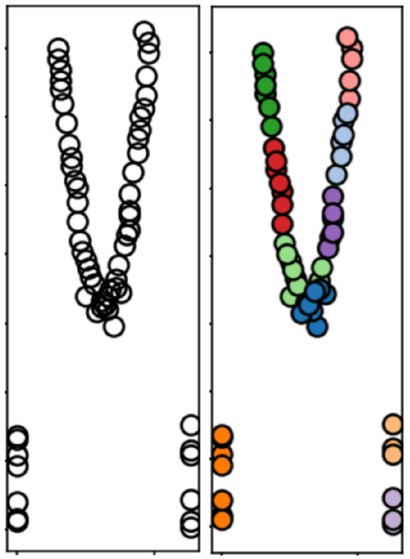
Navier Stokes Simulation Dataset: Probabilistic Map Labeling



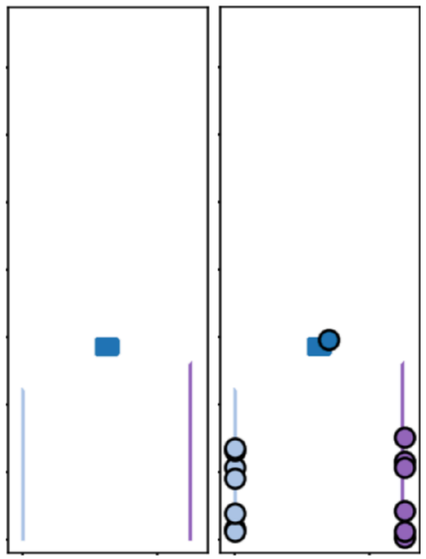
Persistence graph and spaghetti plot



Morse mapping

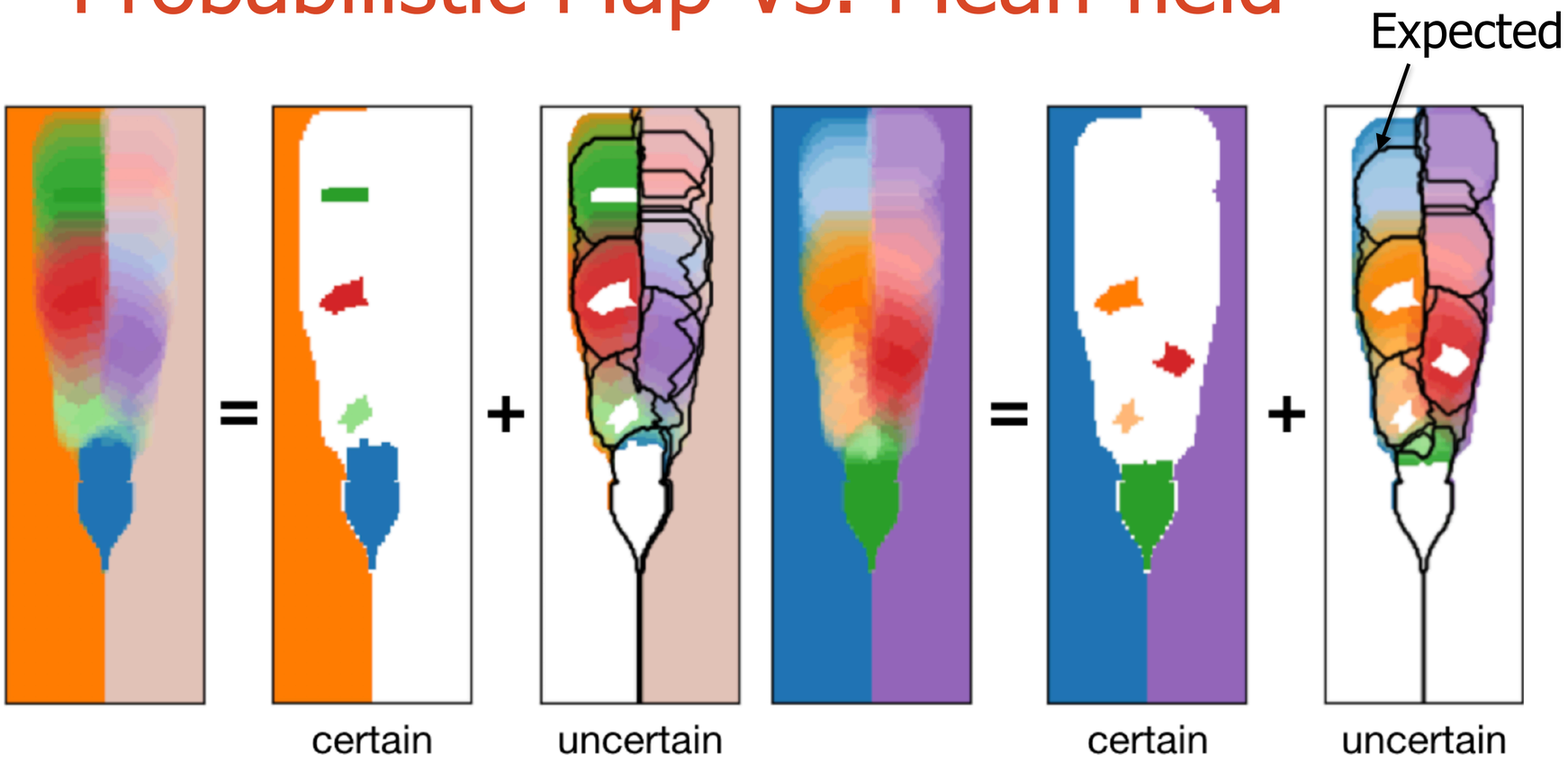


K-means



Nearest mandatory maxima (loose topological information due to large noise levels)

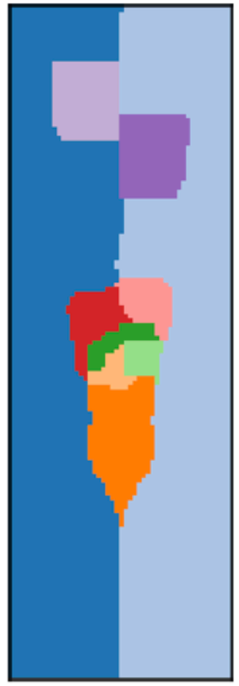
Navier Stokes Simulation Dataset: Probabilistic Map Vs. Mean-field



(a) K-means

(b) Morse mapping

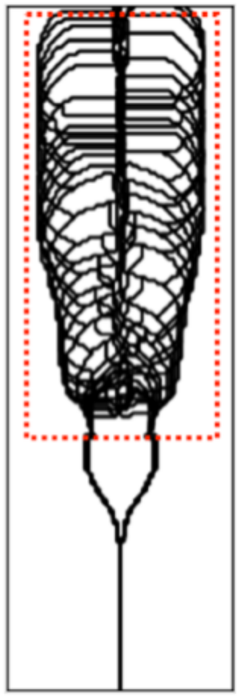
Probabilistic maps



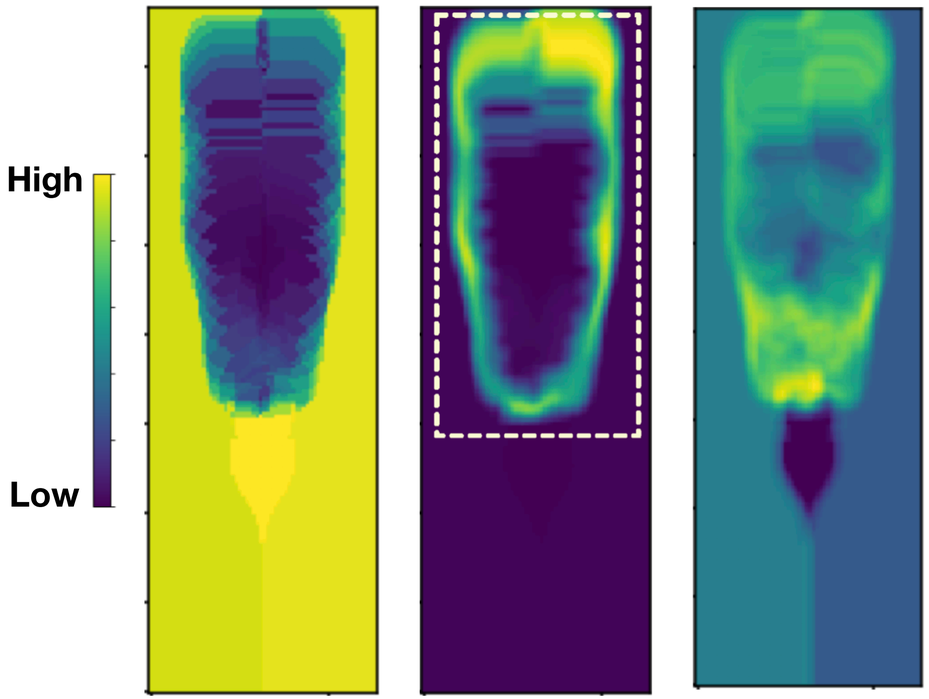
(c) Mean-field

Unreliable 2-cell reconstruction
+ No indication of uncertainty

Navier Stokes Simulation Dataset: Significance Vs. Survival Maps

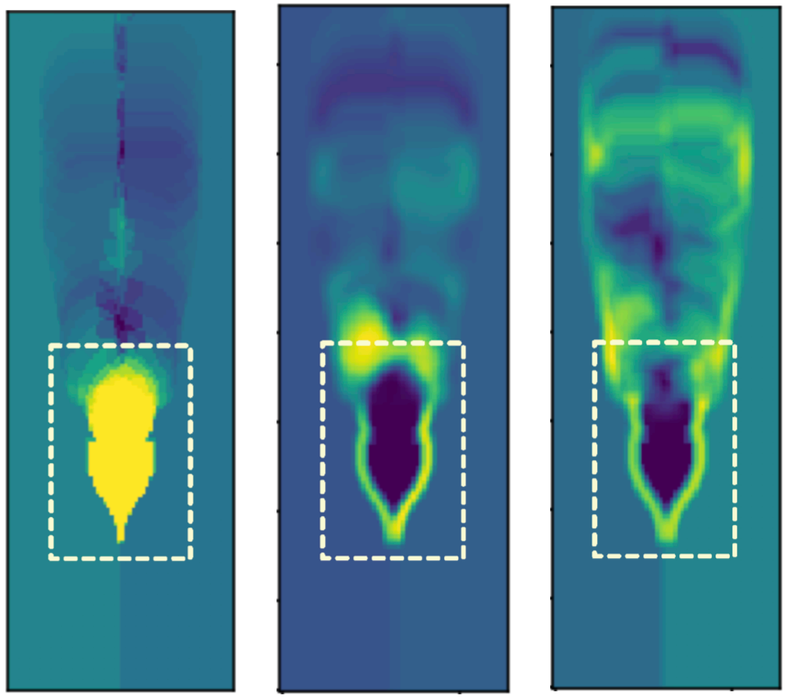


Significance map



Point-wise mean | Point-wise variance | Point-wise entropy

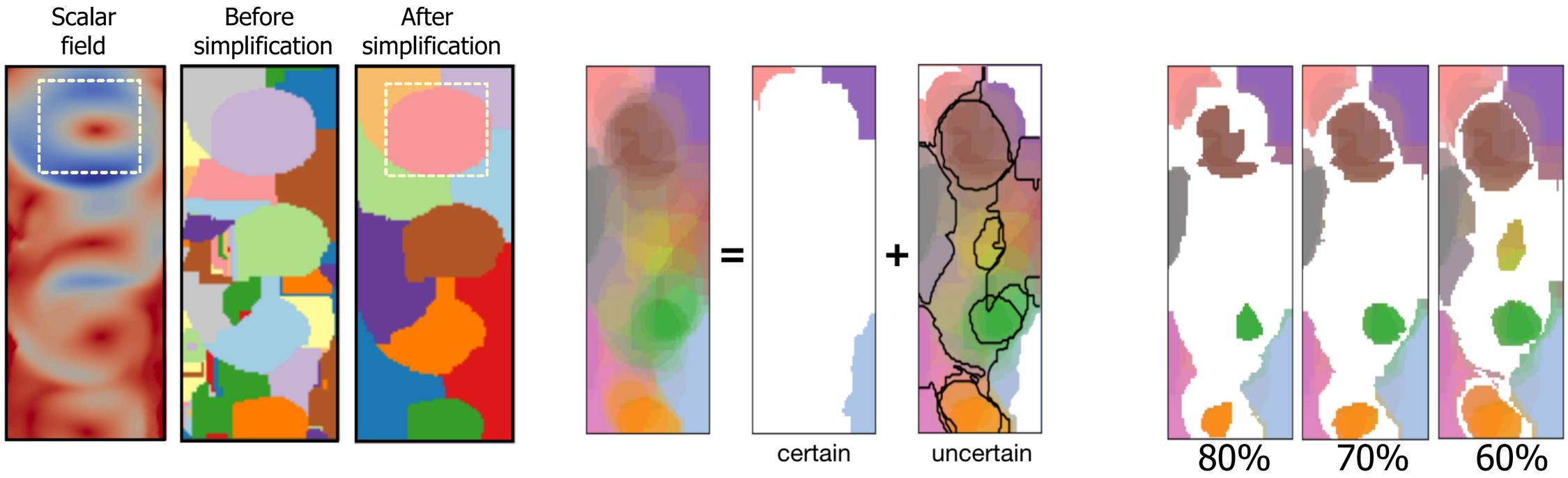
Survival map



Point-wise mean | Point-wise variance | Point-wise entropy

Red Sea Eddy Simulation Dataset

Courtesy: SciVis Contest 2020 dataset (https://kaust-vislab.github.io/SciVis2020/)



Single ensemble member

Probabilistic map

Agreement exploration

Conclusion

- Statistical summary maps (probabilistic, significance, and survival) for structural uncertainty quantification of Morse complexes
- Labeling with mandatory maxima, k-means clustering, and Morse mapping for deriving probabilistic maps
- Improved topological recovery along with uncertainty visualizations using probabilistic maps in comparison to mean-field visualizations
- Color blending, entropy- and variance-based visualizations, interactive probability density queries for visualizing uncertainties

Thank you for your attention!

This research is supported in part by IIS-1910733, DBI-1661375, and IIS-1513616; the NIH grants P41 GM103545-18 and R24 GM136986; the DOE grant DE-FE0031880; and the Intel Graphics and Visualization Institutes of XeLLENCE.

For any questions, please contact me at:
Email: tushar.athawale@gmail.com