Cases for Computation of $cdf_P(p)$

July 31, 2018

In this document, we tabulate the integration cases for computing the cumulative density function $cdf_P(p)$. All integrations depend upon the following two integrals:

$$
\begin{aligned}
I_1 &= \frac{1}{d-c}\int (r-c) * pdf_R(r) \\
I_2 &= \int pdf_R(r), \text{where} \\
pdf_R(r) &= \frac{k^2+p}{(k+r)^2(b-a)}
\end{aligned}
$$

The $cdf_P(p)$ can be computed by setting appropriate limits for the integrals I_1 and I_2 followed by computing the sum of the integrals. ∞ appears in the limits of integration I_2 in a few cases. However, the integration is computable in closed form in all cases since integration I_2 is an inverse function of r, as explained in the manuscript in the last paragraph of section 5.1.2. In the tabulated images, the joint density of random variables Y and R is represented by a support bounded between two horizontal lines. The horizontal lines denote limits of Y, where the lower line is at $Y = c$ and the upper line is at $Y = d$. We use the same coloring scheme and notation as in the manuscript for images in the tables. The blue indicates positions where $Y \leq R$ at any position $R = r$. The overlap of joint density with blue regions is highlighted in brown. The brown area indicates the domain of integration for computing $Pr(Y \leq R) = cdf_P(p)$. We present computations for cases represented by all possible areas highlighted in brown and all possible integration limits.

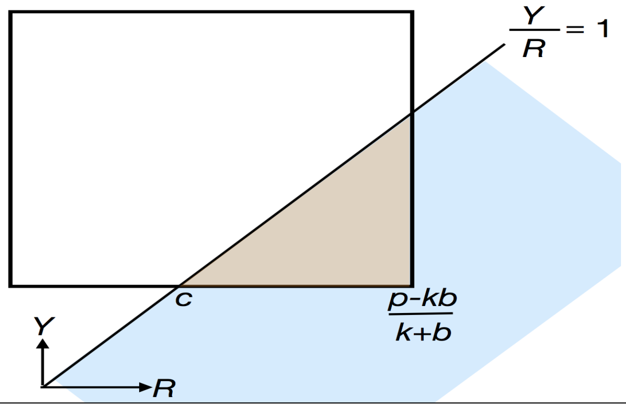
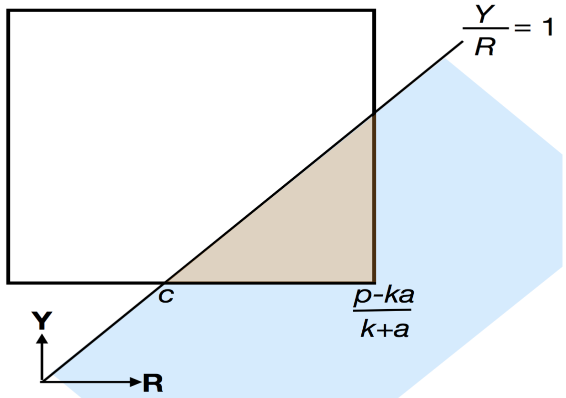
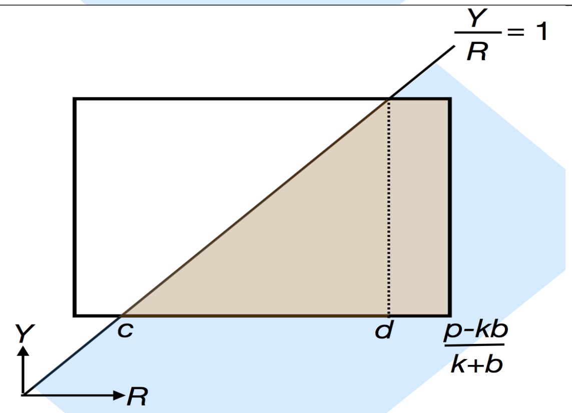
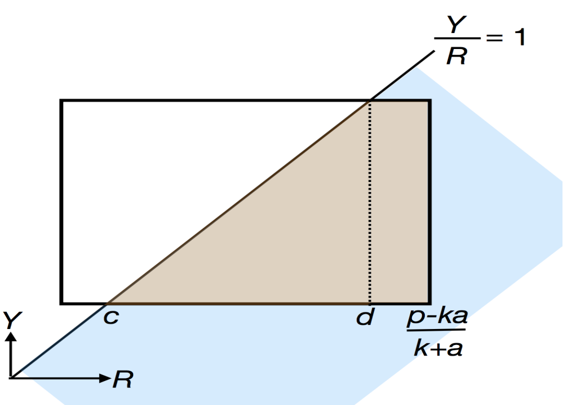
Joint Density of Y and R with respect to $Y/R = 1$	$cdf_P(p)$
	$cdf_P(p) = \int_c^{\frac{p-kb}{k+b}} l_1 dr$
	$cdf_P(p) = \int_c^{\frac{p-ka}{k+a}} l_1 dr$
	$cdf_P(p) = \int_c^d l_1 dr + \int_d^{\frac{p-kb}{k+b}} l_2 dr$
	$cdf_P(p) = \int_c^d l_1 dr + \int_d^{\frac{p-ka}{k+a}} l_2 dr$

Table 1. Finite integration domain cases for computing $cdf_P(p)$.

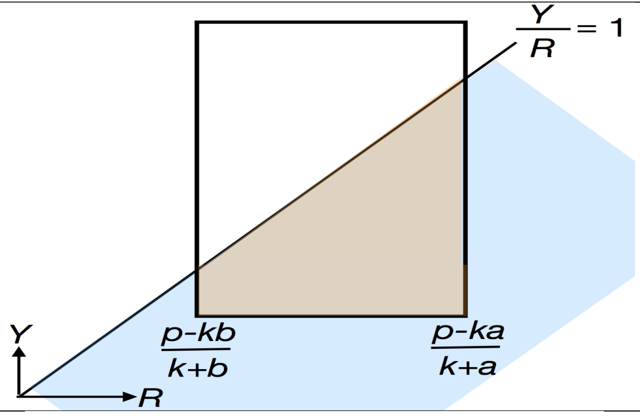
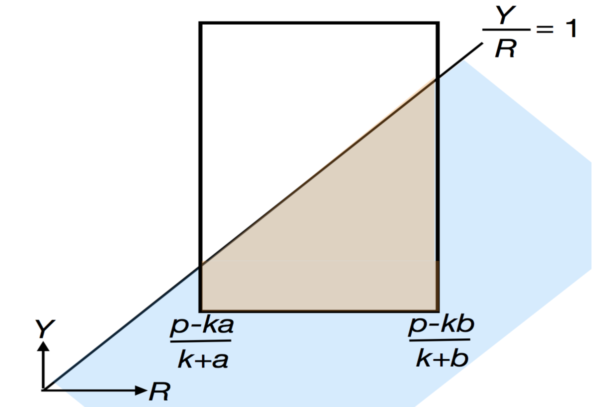
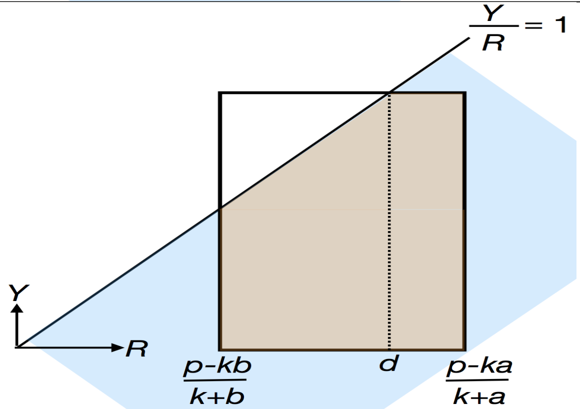
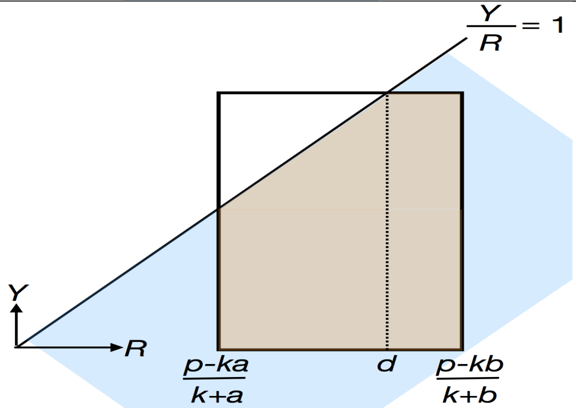
Joint Density of Y and R with respect to $Y/R = 1$	$cdf_P(p)$
	$cdf_P(p) = \int_{\frac{p-kb}{k+b}}^{\frac{p-ka}{k+a}} l_1 dr$
	$cdf_P(p) = \int_{\frac{p-ka}{k+a}}^{\frac{p-kb}{k+b}} l_1 dr$
	$cdf_P(p) = \int_{\frac{p-kb}{k+b}}^{d} l_1 dr + \int_{d}^{\frac{p-ka}{k+a}} l_2 dr$
	$cdf_P(p) = \int_{\frac{p-ka}{k+a}}^{d} l_1 dr + \int_{d}^{\frac{p-kb}{k+b}} l_2 dr$

Table 2. Finite integration domain cases for computing $cdf_P(p)$.

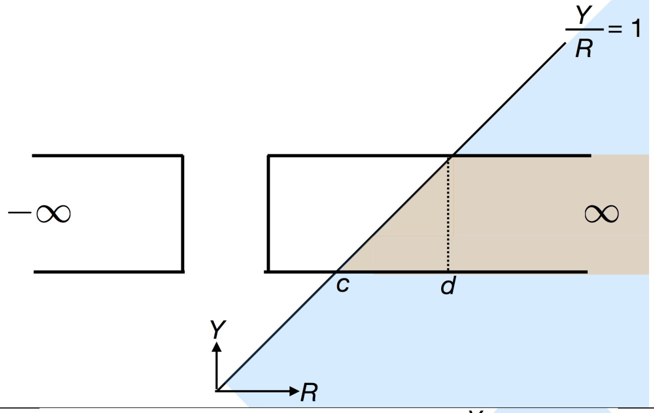
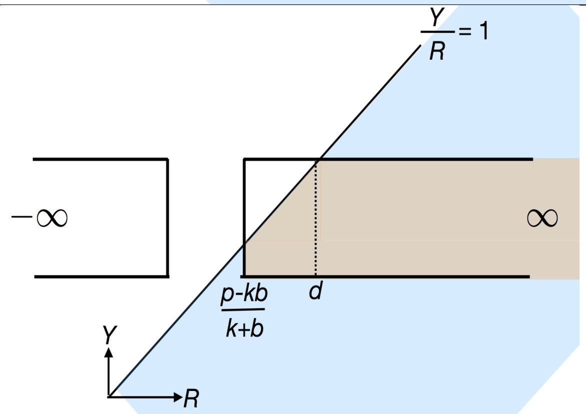
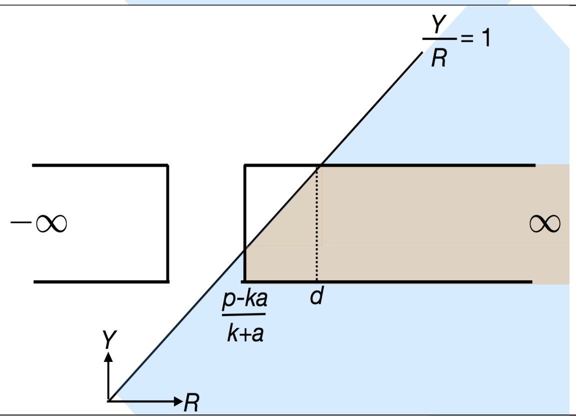
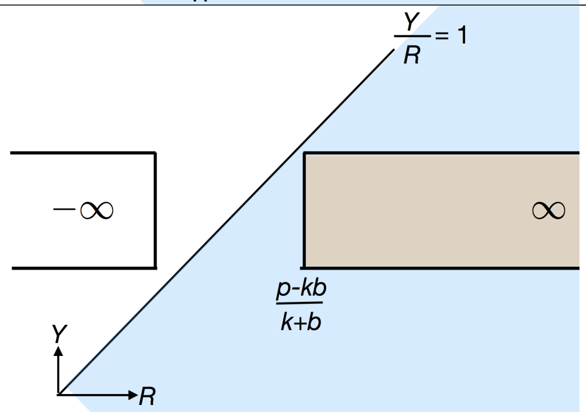
Joint Density of Y and R with respect to $Y/R = 1$	$cdf_P(p)$
	$cdf_P(p) = \int_c^d l_1 dr + \int_d^\infty l_2 dr$
	$cdf_P(p) = \int_{\frac{p-kb}{k+b}}^d l_1 dr + \int_d^\infty l_2 dr$
	$cdf_P(p) = \int_{\frac{p-ka}{k+a}}^d l_1 dr + \int_d^\infty l_2 dr$
	$cdf_P(p) = \int_{\frac{p-kb}{k+b}}^\infty l_2 dr$

Table 3. Infinite integration domain cases for computing $cdf_P(p)$.

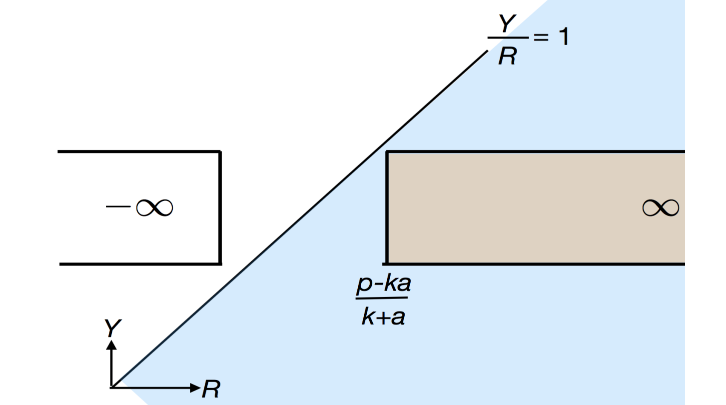
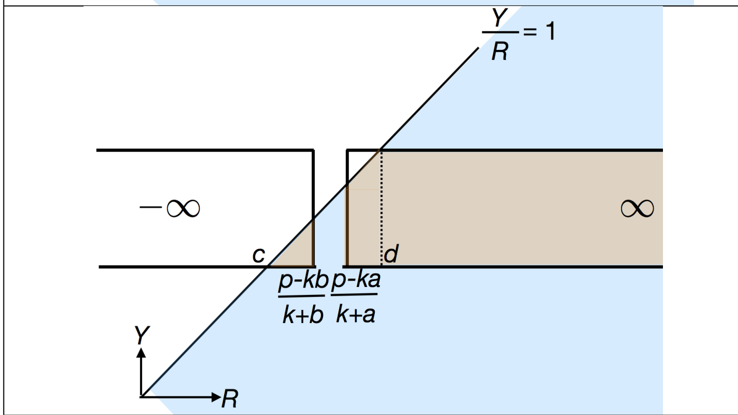
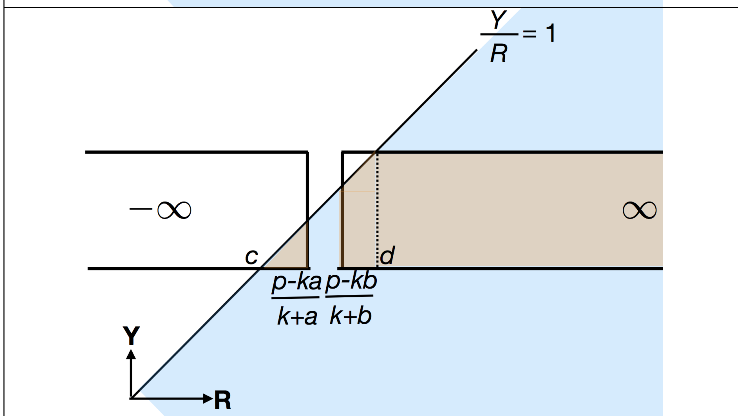
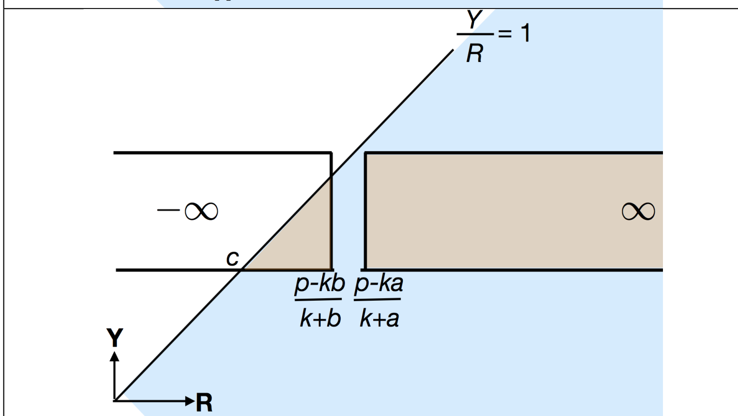
<table>
<tr><th>Joint Density of Y and R with respect to $Y/R = 1$</th><th>$cdf_P(p)$</th></tr>
<tr><td>
</td><td>$cdf_P(p) = \int_{\frac{p-ka}{k+a}}^{\infty} l_2 dr$</td></tr>
<tr><td>
</td><td>$cdf_P(p) = \int_{c}^{\frac{p-kb}{k+b}} l_1 dr + \int_{\frac{p-ka}{k+a}}^{d} l_1 dr + \int_{d}^{\infty} l_2 dr$</td></tr>
<tr><td>
</td><td>$cdf_P(p) = \int_{c}^{\frac{p-ka}{k+a}} l_1 dr + \int_{\frac{p-kb}{k+b}}^{d} l_1 dr + \int_{d}^{\infty} l_2 dr$</td></tr>
<tr><td>
</td><td>$cdf_P(p) = \int_{c}^{\frac{p-kb}{k+b}} l_1 dr + \int_{\frac{p-ka}{k+a}}^{\infty} l_2 dr$</td></tr>
</table>

Table 4. Infinite integration domain cases for computing $cdf_P(p)$.

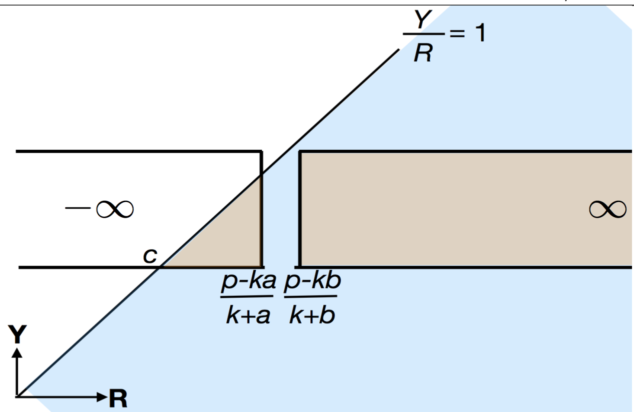
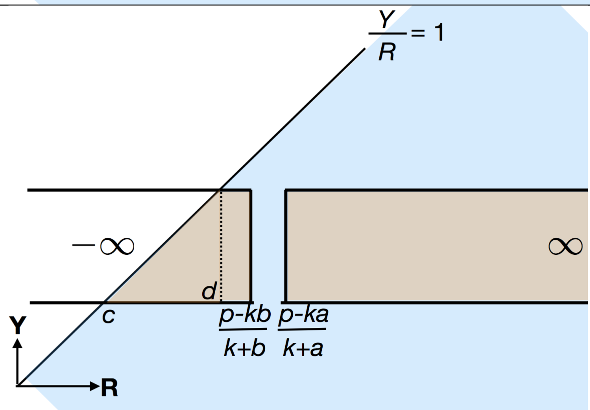
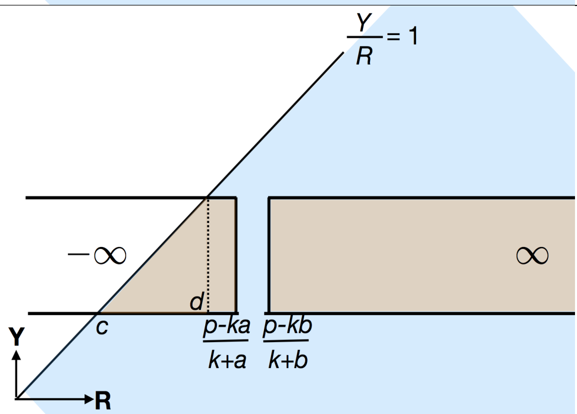
Joint Density of Y and R with respect to $Y/R = 1$	$cdf_P(p)$
	$cdf_P(p) = \int_c^{\frac{p-ka}{k+a}} l_1 dr + \int_{\frac{p-kb}{k+b}}^{\infty} l_2 dr$
	$cdf_P(p) = \int_c^{d} l_1 dr + \int_d^{\frac{p-kb}{k+b}} l_2 dr + \int_{\frac{p-ka}{k+a}}^{\infty} l_2 dr$
	$cdf_P(p) = \int_c^{d} l_1 dr + \int_d^{\frac{p-ka}{k+a}} l_2 dr + \int_{\frac{p-kb}{k+b}}^{\infty} l_2 dr$

Table 5. Infinite integration domain cases for computing $cdf_P(p)$.